April 6, 2026 - Monday

Direct Linearising Transform and Lagrange structures for KP type equations

Frank Nijhoff
University of Leeds

Direct linearisation has been successful as a method of finding novel integrable equations and their interrelations. The direct linearising transform (DLT) is generalisation that maps possibly non-free solutions to new solutions via linear integral equations. In particular various (continuous, fully and semi-discrete) families of KP type equations can be treated through this approach, and has led to novel classes of solutions. A particular application is the DLT for the τ-function of the KP system. I aim also to discuss Lagrange multiform aspects of the KP systems, and reductions such as to the Gel'fand-Dikii family of lattice equations.

Three-parameter families of integrable difference equations and associated bond systems

Maciej Nieszporski
University of Warsaw

There is an observation in a seminal article by Nijfoff et al. [1] that integrable difference equations appear in triplets. I will report on the state of research concerning three-parameter families of integrable difference equations starting from articles $[2, 3, 4]$ where the families were associated to difference systems defined on edges (bonds) of a lattice.

References

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[2] P. Kassotakis and M. Nieszporski. Families of integrable equations. SIGMA 7 (2011), Art. No. 100.

[3] P. Kassotakis and M. Nieszporski. On non-multiaffine consistent around the cube lattice equations. Physics Letters A 376, no. 45 (2012): 3135C40.

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Toda lattice hierarchy: Old and new

Di Yang
University of Science and Technology of China

The Toda lattice hierarchy, aka the infinite Toda chain hierarchy, is an important integrable hierarchy that has many connections to various areas of mathematics and physics, in particular to enumerative geometry. In this talk, we review some of its known applications and present new applications.

Height growth and Diophantine integrability

Rod Halburd
University College London

The (logarithmic) height of a non-zero rational number a/b, where a and b are co-prime integers, is ln $\max\{|a|, |b|\}$. It is a natural measure of the complexity of the rational number. For rational discrete equations with solutions that are sequences of rational numbers, an estimation of the height growth of solutions is perhaps the quickest and easiest "test" of integrability to implement on a computer. In this talk I will describe heights on number fields and prove precise estimates for the height of iterates for some discrete Painlevé equations. This is joint work with Jilong Zhang, Beihang University.

Geometric structure of thermodynamic evolution and integrable one-forms

Sikarin Yoo-Kong
Naresuan University

Thermodynamic processes can be geometrically interpreted in a manner analogous to Hamiltonian evolution on phase space. Although thermodynamics and Hamiltonian mechanics appear mathematically different, they share a common geometric structure formulated in terms of differential one-forms. From this perspective, the path independence of thermodynamic processes can be expressed as the integrable one-form condition. This work explores the geometric formulation of thermodynamic evolution and demonstrates how the integrability of the associated one-form naturally characterises reversible processes.

Discretisation of a symmetry algebra: the Calogero-Moster model case

Giorgio Gubbiotti
Università degli Studi di Milano & INFN Sezione di Milano

Stimulated by our previous work on discretisation of the harmonic oscillators [P. Drozdov et al, 2025 Phys. Scr. 100 095228] and their symmetry algebra, we determine the complete structure of the symmetry algebras associated with the N-body Calogero-Moser system and its maximally superintegrable discretisation obtained by Nijhoff and Pang [F. W. Nijhoff and G.-D. Pang, 1994 Phys. Lett. A 191 101-107]. We prove that, differently from the previously known examples, the discretisation naturally leads to a nontrivial deformation of the continuous symmetry algebra, with the discretization parameter playing the role of a deformation parameter. This phenomenon shows how discrete superintegrable systems can be source of deformed polynomial algebraic structures.

This is a joint work with P. Drozdov (Università degli Studi di Udine & INFN Sezione di Trieste)

and D. Latini (Università degli Studi di Milano & INFN Sezione di Milano).

A many-body McMillan integrable symplectic map

Ruguang Zhou
Jiangsu Normal University

The McMillan map, originally proposed by Edwin McMillan in 1971, is a fundamental example of a nonlinear integrable mapping in dynamical systems. It has been widely studied in both mathematics and physics due to its elegant mathematical structure and its relevance to practical applications, notably in accelerator physics.

In this talk, we introduce and investigate a novel many-body generalization of the McMillan map that is integrable and symplectic. We demonstrate that this map is intimately connected to the Kaup-Newell hierarchy; in particular, it arises as a Bäcklund transformation for the restricted Kaup-Newell flows and shares their integrals of motion. By introducing root variables and employing Abel-Jacobi coordinates, we show that the discrete-time evolution corresponds to a shift on the associated Jacobi variety.

April 7, 2026 - Tuesday

Integrability, nonintegrability, and Darboux polynomials

Reinout Quispel
La Trobe University

In this talk I plan to commence with some reminiscences of more than 40 years of scientific collaboration with Frank Nijhoff.

This will be followed by a discussion of Darboux polynomials for continuous systems, and a discussion of the Kahan discretisation, which maps quadratic differential equations to difference equations, and also present some novel discoveries.

Integrable Lotka-Volterra systems

Peter van der Kamp
La Trobe University

Lotka-Volterra systems are ordinary differential equations of the form

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = x_i \left(r_i + \sum_{j=1}^{n} A_{ij} x_j \right), \quad i = 1, 2, \ldots, n,$$

where the vector $\mathbf{r}$ and matrix $\mathbf{A}$ do not depend on $\mathbf{x}, t$. Using the theory of Darboux Polynomials we identify many multi-parameter families of integrable Lotka-Volterra systems.

Commutant constructions, superintegrability and deformations of Lotka-Volterra systems

Ian Marquette
La Trobe University

The Lie-algebraic notion of commutant was developed and applied in context of superintegrable systems, missing label problems, dynamical and hidden symmetries and subalgebra chains in nuclear physics $[1, 2, 3]$. Those constructions involve polynomials in the universal enveloping algebra of a Lie algebra. In context of superintegrability this allows to build algebraic Hamiltonians, integrals and symmetry algebra. An important case relies classical Lie algebras and their Cartan subalgebra. One of the main discoveries, is that commutants lead to the notion of polynomial algebras i.e. algebraic structures in which the commutators of generators relate to polynomial expressions of the generators. One main advantage of such approach for superintegrable systems is that it does not use apriori explicit realizations. This framework has also been related to different geometric constructions [4].

In this talk will also consider the notion of commutant in the setting of Poisson algebras in context of certain Lotka-volterra systems [5]. The approach taken here is different from previous works as the commutant is used to deform a superintegrable Hamiltonian differential equations. By taking a subalgebra of the algebra of integrals, and considering the set of functions that Poisson commute with that subalgebra, the Hamiltonian can be deformed while retaining integrability or superintegrability. We deform Liouville integrable and superintegrable Lotka-Volterra systems. We present different explicit constructions considering Abelian and non-Abelian symmetry subalgebras. In previous work related to semi simple Lie algebras and their Poisson-Lie algebras, the elements of the commutant were polynomials, in this context the commutant will be rational and determined via solving systems of partial differential equations. We obtain superintegrable systems for specific dimensions, and in arbitrarily dimensions. This talk is mainly based on recent results [5].

References

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[2] R Campoamor-Stursberg ,D Latini, I Marquette, J Zhang, YZ Zhang, On the construction of polynomial Poisson algebras: a novel grading approach, Eur. Phys. J. Plus 141, 155 (2026)

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[4] K Jiang, G Ma, I Marquette, J Zhang, YZ Zhang, Poisson Centralisers and Polynomial Superintegrability for Magnetic Geodesic Flows on Reductive Homogeneous Spaces, arXiv:2601.01369

[5] Ian Marquette, P. H. van der Kamp and G.R.W Quispel, Subalgebras of integrals, commutants, and superintegrable deformations of Lotka-Volterra systems, arXiv:2602.17945

Approximate symmetry approach for differential-difference equations

Jing Ping Wang
Ningbo University

In this talk, we discuss the approximate symmetry approach developed for differential-difference equations. This framework allows us to derive necessary conditions for integrability, to test whether a given equation is integrable, and to advance the classification of integrable equations. We apply the formalism to solve the classification problem for anti-symmetric scalar quasi-linear equations, covering both commutative and noncommutative cases. Among the equations obtained, several are new. This is joint work with A.V. Mikhailov and V.S. Novikov, recently published in Communications in Mathematical Physics.

Difference Hamiltonian operators and multiplicative Poisson vertex algebras

Matteo Casati
Ningbo University

I will present some recent results on the classification of Hamiltonian difference operators, obtained within the framework of the theory of multiplicative Poisson vertex algebras (MPVAs). In the seminal paper [1], scalar operators have been classified up to order $(-5, 5)$; after reviewing the notion of MPVA, I will present the results of [2], that extends the classification to one-dimensional, two-component operators, and the recent multidimensional extension developed in [3].

References

[1] A. De Sole, V. G. Kac, D. Valeri, M. Wakimoto. Poisson λ Brackets for Differential-Difference Equations. IMRN, 13, 2020.

[2] M. Casati, D. Valeri. Multi-component Hamiltonian difference operators. Nonlinearity 38, 2025.

[3] P. Yang, M. Casati. Multidimensional multiplicative Poisson vertex algebras. Preprint arXiv:2512.06704, under review.

Discrete Painlevé equations with constraints (Part I): Their geometry (joint work with A. Dzhamay, Y. Shi, and R. Willox)

Alexander Stokes
Waseda University

Discrete Painlevé equations are discrete dynamical systems on families of Sakai surfaces that admit actions of extended affine Weyl groups. The Painlevé dynamics is generated by the actions of translations or quasi-translations. The Sakai classification of discrete Painlevé equations is complete on the level of surfaces, as well as on the level of the symmetry groups that generate the dynamics in the generic case, i.e. when the family exhausts the moduli of surfaces of that type. However, there are non-generic cases, i.e. discrete Painlevé equations with constraints, in which the symmetry group forms a subgroup of the generic one for its type, and does not appear explicitly in the classification. We describe the geometric origin of some such non-generic surfaces and present a geometric method to construct examples with interesting symmetry groups, including $\widetilde{W}\left(2G_2^{(1)}\right)$ and $W\left(C_4^{(1)}\right)$.

April 8, 2026 -Wednesday

Stratified superintegrable systems

Nicolai Reshetikhin
Tsinghua University

In superintegtable systems singularities of Liouville tori can be more interesting. In this talk it will be demonstrated on the example of spin Calogero-Moser-Sutherland system. A more general structural discussion of integrable systems on stratified symplectic spaces will be presented. The talk is based on a joint work with K. Jiang, H. Xiao, and C. Zhuo.

Cluster algebras and q-Painlevé equations

Teruhisa Tsuda
Aoyama Gakuin University

A cluster algebra is an algebraic structure generated by operations of a quiver called the mutations and their associated simple birational mappings. We present a systematic way to derive a birational representation of Weyl groups from cluster algebras by means of graph-combinatorial point of view. This framework covers a broad class of Dynkin diagrams and in affine case leads to the q-Painlevé equations and their higher-order extensions. By using the normal form of a skewsymmetric integer matrix, Darboux coordinates can be selected while preserving birationality of the dynamical systems. The (multiplicative) root variables naturally emerges from the Casimir functions of the Poisson structure of cluster algebras. If time permitted, we also discuss the τ-function formalisms.

This talk is based on the collaborative research:

[1] T. Masuda, N. Okubo & TT: Birational Weyl group actions and q-Painlevé equations via mutation combinatorics in cluster algebras, arXiv:2303.06704.

[2] T. Masuda, Y. Mizuno, N. Okubo, Y. Terashima & TT: -, Part II: τ-function formalism (a tentative title), in preparation.

Towards a formulation of the singular pattern approach for degree growth in higher-dimensional birational systems

Tomoyuki Takenawa
Tokyo University of Marine Science and Technology

For a birational map f, let $d_n = \deg(f^n)$ denote the degree of its n-th iterate. The quantity $e(f) = \lim_{n\to\infty} \frac{1}{n} \log(d_n)$ is called the algebraic entropy, and is a fundamental measure of the complexity of a birational dynamical system. In this talk, we consider methods for determining the degree growth.

Several approaches to degree growth are known, including algebro-geometric constructions of spaces of initial conditions and detailed analyses of factorisation patterns of rational functions. While these methods are powerful, they are often technically involved and computationally costly. In contrast, Halburd proposed a simple method for single-variable higher-order recurrences: by observing singularity patterns in the sense of Grammaticos-Ramani, one can directly derive a recurrence relation satisfied by the degree sequence. However, this method is difficult to apply when the resulting relations are trivial.

In this talk, we reconsider Halburd's idea from the viewpoint of higher-dimensional birational dynamics. This perspective suggests the possibility of extending the method to a broader class of examples that cannot be handled within the original one-variable framework. At the same time, such an extension requires a more rigorous treatment of notions that have not yet been defined sufficiently clearly, such as push-forwards and multiplicities associated with singularity patterns. We therefore discuss how these notions should be defined in this framework, as a step toward formulating the singular pattern approach for N-dimensional birational maps.

April 9, 2026 -Thursday

Singularity structure of Poincaré functions and some special solutions of functional differential equations

Techheang Meng
ITC, Cambodia

The difference equation $y(z+1) = R(y(z))$, where R is a rational function, is integrable only if R has no attracting fixed points. When R has an attracting fixed point, the local solution there can be constructed using Poincaré functions, satisfying $g(\lambda z) = R(g(z))$, where $0 < |\lambda| < 1$. When R is quadratic, the Poincare function turns out to be "nice" in the sense that its singularity structure can be determined explicitly. However, when the degree of R is at least three, there are new various features to the singularity structures and we will investigate this through several examples. At the end of the talk, we will generalize the arguments to certain firstorder functional differential equations.

Aspects of lattice Boussinesq equations: generalized 3D consistency and lattice BSQ-Q3 system

Pengyu Sun
Shanghai University

In this talk, I will introduce two new results concerning lattice Boussinesq (BSQ) equations, which was a collaboration with Cheng Zhang and Frank Nijhoff. First, we show that the lattice potential BSQ equation, defined on a nine-point square lattice, admits a natural extension of three-dimensional consistency to a $3 \times 3 \times 3$ cube-a cubic sublattice consisting of 27 vertices. Second, we construct a new 3Dconsistent three-component system, which we refer to as the lattice BSQ-Q3 system, serving as the BSQ analogue of the Q3(δ) equation in the Adler-Bobenko-Suris classification. The system is derived via a discrete gauge transformation between two Lax systems of lpBSQ, with the parameter δ arising from a GL_3 action. In a degeneration limit, it reduces to a PGL_3-invariant integrable lattice equation generalising the PGL_2-invariant Schwarzian BSQ equation.

Elliptic solitons and Painlevé IV solitons of AKNS/NLS systems

SenYue Lou
Ningbo University

In this talk, we first develop a group-invariant decomposition approach, and then investigate elliptic solitons using space-time translations and nonlocal symmetries, followed by an analysis of Painlevé IV solitons via scaling and residual symmetries.

Different from the known elliptic solitons, we propose more general elliptic solitons with two additional free parameters for periodic elliptic waves. Some types of elliptic solitons are valid for both the AKNS systems and the NLS equations, while others hold only for the AKNS system but not for the NLS equations.

Regarding the Painlevé IV solitons, we find that the key ODE reduction equation is related to the Painlevé IV transcendent only for a complex argument. Consequently, the general Painlevé IV solitons are valid only for the AKNS system and not for the NLS equation. To obtain Painlevé IV solitons for the NLS equations, one must consider real solutions of the complex Painlevé IV equation. In this talk, instead, we study Painlevé IV solitons for the AKNS/NLS systems by investigating solutions of a variant of the Painlevé IV equation

$$2VV_{\eta\eta\eta} + 16\omega\eta VV_\eta - 3V_{\eta\eta}^2 - \left(\eta^2 - 2ib_1\right)V_\eta^2 - 48\omega^2V^2 = 0$$

with real argument η and two free constants $\{\omega, b_1\}$.

In particular, for the rational solutions of this variant Painlevé IV equation, we define three novel types of polynomials via some semi-discrete systems-or equivalently, recursion relations, say,

$$s_{n+1}s_{n-1} = \left[\eta^4 + 45(3n+2)^2\right]\eta^2 s_n^2 - 54\eta^3 s_n s_{n\eta} + 324(3n+2)^2 s_n s_{n\eta\eta} - 81(6n+1)(6n+7)s_{n\eta}^2.$$

Although the solutions of the variant Painlevé IV equation are taken as rational, the corresponding solutions of the AKNS/NLS systems may be either rational or irrational.

Some New Results on Integrable Benjamin-Ono type Equations

Xing-Biao Hu
AMSS, Chinese Academy of Sciences

In the talk, I will first give a very brief introduction to relevant study on integrable integro-differential equations involving Hilbert operator and then show you some recent results on BO-type equations.This is a joint work with Yingnan Zhang, Lingjuan Yan, Yajie Liu and Gegenhasi.

Linear differential and difference equations with apparent singularities

Oleg Chalykh
University of Leeds

Consider a linear ODE in the complex domain,

$$y^{(n)} + a_1(x)y^{(n-1)} + \cdots + a_n(x)y = 0, \quad y = y(x).$$

Its singular point $x = x_0 \in \mathbb{P}^1$ is called an apparent singuarity if all solutons $y(x)$ are meromorphic near this point. Equivalently, $x = x_0$ is a regular singular point and the monodromy around x_0 is trivial.

What can one say about such an equation if all its finite singular points are apparent singularities? And what if, in addition, this property holds for the equation

$$y^{(n)} + a_1(x)y^{(n-1)} + \cdots + a_n(x)y = \lambda y \quad \text{for arbitrary } \lambda \in \mathbb{C}?$$

I will give an answer for the case when all coefficients $a_i(x)$ are rational functions, bounded at

$x = \infty$. I will then discuss an analogous problem for difference equations.

Pfaffians as τ-functions of the BKP hierarchy: a constructive parametrization of complex pure spinors de E. Cartan

Yuancheng Xie
Shenzhen MSU-BIT University

It is well known that τ-functions of KP hierarchy are parameterized by points in Sato's Universal Grassmannian manifold (UGM). These τ-functions have Schur expansions with coefficients satisfying Plücker relations. In this talk we will show that all τ-functions of BKP hierarchy can be written as Pfaffians of skew-symmetric matrices. These τ-functions are parameterized by points in the universal orthogonal Grassmannian manifold (UOGM). They have natural Schur-Q expansions with coefficients satisfying Cartan-Plücker relations. As a byproduct this parameterization gives a constructive description for complex pure spinors de E. Cartan. As an application, we reprove a theorem due to A. Alexandrov which states that τ-functions of KdV satisfy BKP up to rescaling of the time parameters by 2 . We prove this by showing that the KdV hierarchy can be viewed as 4-reduction of the BKP hierarchy. This interpretation gives complete characterization for the KdV orbits inside the BKP hierarchy. This talk is based on preprint arXiv:2210.03307.

On initial value problems for a class of three-term Gale-Robinson recurrences

Xiang-Ke Chang
Academy of Mathematics and Systems Science, Chinese Academy of Sciences

We will talk about a class of three-term Gale-Robinson recurrences, including Somos-4 and Somos-5, that can be obtained as reductions of the discrete KP equation. It is known that they exhibit interesting integrality, behind which it is the Laurent phenomenon appearing as a key property of cluster variables in Fomin and Zelevinsky's cluster algebras. Nevertheless, our intention is to propose a unified approach to solve initial value problems for this class of three-term Gale-Robinson recurrences based on their Lax pairs. With the help of hyperelliptic curves and continued fractions, we obtain the explicit solutions in terms of Hankel determinants, from which the Laurent properties straightforwardly follow.

Lagrangian formulation of the Darboux system

Maksim Pavlov
Shandong University of Science and Technology

The classical Darboux system governing rotation coefficients of threedimensional metrics of diagonal curvature possesses an equivalent formulation as a sixth-order PDE for a scalar potential (related to the corresponding tau-function). We demonstrate that this PDE is Lagrangian and can be viewed as an explicit scalar form of the 'generating PDE of the KP hierarchy' as discussed recently in Nijhoff [arXiv:2406.13423] in the Lagrangian multiform approach to the Darboux and KP hierarchies. Scalar Lagrangian formulations for differential-difference and fully discrete versions of the Darboux system are also constructed. In the first three cases (continuous and differential-difference with one and two discrete variables), the corresponding Lagrangians are expressible via elementary functions (logarithms), whereas the fully discrete case requires special functions (dilogarithms).

April 10, 2026 -Friday

Integrability paradigm inspired by the Yang-Baxter equation

Jarmo Hietarinta
University of Turku

Consider three quantum particles (with different velocities) moving on a line. The order in which they collide depends on their initial positions, but if the Yang-Baxter equation is satisfied, the final result does not depend on the order. In this talk I will show that this basic idea can be observed in many different situations, if we use a creative interpretation of "particles" and "scattering". The canonical example leading to the Yang-Baxter equation has an immediate natural extension to the set theoretical case, as exemplified by Yang-Baxter maps. However, this paradigm can also be observed in the "Consistency-Around-a-Cube" concept for quad equations on a $\mathbb{Z}^2$ lattice, as well as in the three-soliton condition in Hirota's bilinear formalism.

On multi-component pentagon maps

Pavlos Kassotakis
University of Patras

In this talk, first, we review the connection of pentagon with Yang-Baxter and tetrahedron maps. Then, we propose a construction that produces families of multi-component maps from a single given map. When the given map is a pentagon map, we obtain multi-component pentagon and entwining pentagon maps.