Performance Analysis of Distributed State Estimation under Misspecified Noise Covariances

Xiaoxu Lyu(吕晓旭)
The Hong Kong University of Science and Technology

This report thoroughly discusses the performance of a consensus-based distributed filter when noise covariances are inaccurately specified. Initially, we define four key metrics: the nominal filter parameter, the nominal estimation error covariance, the ideal filter parameter, and the ideal estimation error covariance. We formulate expressions to capture the disparities between these metrics and establish their one-step interrelations. These relationships illustrate the degradation in performance due to incorrect noise covariance specifications and clarify how to evaluate the estimation error covariance using the nominal filter parameter. We emphasize the influence of the number of information fusion steps on these relationships. Additionally, we extend the one-step findings to develop recursive relationships. We then prove the convergence of these metrics under the condition of collective observability, demonstrating that the convergence of the nominal filter parameter ensures the convergence of the estimation error covariance. Moreover, we establish bounds on the estimation error covariance under misspecified noise covariances by leveraging the Frobenius norms of the noise covariance deviations and the trace of the nominal filter parameter. Additionally, we analyze the performance of distributed filtering for continuous-time systems.

The Gaussian Mixture Optimal Transport Ensemble Kalman Filter and Its Applications to Partially Linear Systems

Xue Luo(罗雪)
Beihang University

In this talk, we revisit the Gaussian mixture optimal transport Ensemble Kalman filter (GM-OT-EnKF) to make it more efficient in some partially linear problems. We first show the equivalence between the OT-EnKF with the classical Kalman filter (KF) in the linear system with Gaussian initial distribution. Next, we compare the Gaussian sum filter (GSF) with the GM-OT-EnKF in the setting of linear system with GM initial distribution. The updating of the components' weights in GM-OT-EnKF is finer than those in the GSF, due to the flexibility induced by the particles. These observations in the linear system suggest two adaptions to the original GM-OT-EnKF corresponding to partially linear systems. The one is when the state's equation is linear, the EM algorithm is unnecessary in every cycles; the other one is when the observation is linear, the posterior mean and covariance matrix should be updated explicitly, rather than the empirical ones. The GM-OT-EnKF with either one of the two adaptions above is called the compact GM-OT-EnKF in this paper. The efficiency and accuracy of our proposed algorithm have been numerically verified in the estimation of the states in the Lorenz 63 system and the prediction of the remaining useful life of the lithium-ion batteries.

Deep Learning Solutions to Nonlinear Filtering Problems: A Recurrent Neural Network Perspective

Xiuqiong Chen(陈秀琼)
Renmin University of China

The filtering problem of estimating the state of a stochastic dynamical system from noisy observations is of central importance in engineering, and high-dimensional nonlinear filtering is still a challenging problem. This problem is reduced to solving the Duncan-Mortensen-Zakai (DMZ) equation which is satisfied by the unnormalized conditional density of the state given the observation history. For general nonlinear filtering problems, we leverage on the representation ability of recurrent neural network and provide a computationally efficient and optimal framework for nonlinear filter design based on Yau-Yau algorithm and recurrent neural network.

Recent progress on the Brockett-Mitter program on finite-dimensional filter

Xiaopei Jiao(焦小沛)
BIMSA

In this talk, we will present and summarize recent advances in the study of finite-dimensional filters, with a particular focus on cases where the estimation algebra has non-maximal rank. Since Roger Brockett and Sanjoy Mitter introduced the concept of estimation algebra at the 1983 International Congress of Mathematicians, the nonlinear filtering community has shown sustained interest in identifying conditions under which the conditional density can be described using a finite set of statistics.

The development of finite-dimensional filters (FDFs) has played a central role in nonlinear filtering theory. Two fundamental challenges in this area are the classification of such filters and the discovery of new types. Around the year 2000, Stephen Yau and collaborators for the first time successfully completed the classification of FDFs associated with estimation algebras of maximal rank. This significant result resolved a long-standing question posed by the school of Brockett and Mitter, using a combination of advanced analytical techniques.

Since then, attention has turned toward the more complex case of non-maximal rank estimation algebras. Through continued effort and collaboration across the field, we have now achieved a partial resolution of the Mitter conjecture in arbitrary state-space dimensions. This line of work has introduced novel mathematical tools and opened promising directions for future research aimed at a complete classification of finite-dimensional filters.

Learning an Optimal Investment Policy with Transaction Costs via a Randomized Dynkin Game

Min Dai(戴民)
The Hong Kong Polytechnic University

We study a dynamic investment problem in which an investor seeks to maximize his expected log-

return in the presence of transaction costs and unknown market environments. We show that this problem can be reformulated as an equivalent Dynkin game problem with observable stochastic processes. We introduce an exploratory variant of the Dynkin game by randomizing stopping times through the use of mixed strategies and incorporating an entropy-regularization term. Building on this framework, we develop an interpretable reinforcement learning algorithm tailored for the exploratory Dynkin game. Theoretical analysis, numerical experiments, and empirical tests are provided to demonstrate the effectiveness of the proposed algorithm. This work is in collaboration with Yuchao Dong and Zhichao Lu.

Brownian and Poisson Bridges: Application to Nonlinear Filtering Problems and Error Estimates

Wenhui Dong(董文慧)
Guangxi University

The nonlinear filtering (NLF) problems described by the stochastic systems with jump diffusive state/observation processes have been attracted more and more attentions. Recently, we consider the NLF problem modeled by a diffusive state process with the mixed observations and the correlated noises. One of the observation processes is driven by the Brownian motion correlated with the state process, and the other one is an independent Poisson point process. The state's unnormalized density conditioned on the continuous observation history is described by the Zakai equation. However, in whatever algorithm, the unnormalized density conditioned only on the sub-filtration generated by the discretized observations can be implemented to approximate the solution of the Zakai equation. The main contribution of this work is that we show under certain conditions the mean square error of this approximation is no more than the order , where is the time step, by the technique of Brownian and Poisson bridges. To verify this theoretical convergence rate, we extend the Yau-Yau filtering algorithm originally proposed for the classical NLF problems, to those with the mixed observations and the correlated noises. This algorithm is numerically experimented in the modified cubic sensor problem, which can achieve the error of the order . Moreover, we compare this algorithm with the sampling importance and resampling (SIR) particle filter to illustrate the superiority of the on- and off-line algorithm in both accuracy and efficiency.

The Applications of Yau-Yau Algorithm on McKean-Vlasov Filtering Problem

Zeju Sun(孙泽钜)
Beijing Institute of Mathematical Sciences and Applications(BIMSA)

The McKean-Vlasov filtering problem is a special kind of filtering problem, with the state and/or observation processes governed by McKean-Vlasov stochastic differential equations, which has extensive applications in various scenarios. In this talk, I will present a novel numerical algorithm to solve the McKean-Vlasov filtering problem under the framework of Yau-Yau algorithm. As the first approach to numerically solving the Duncan-Mortensen-Zakai equation associated with the McKean-Vlasov filtering problem, the proposed algorithm can provide accurate estimations of the conditional expectation and conditional probability density of the state process with a reasonable online computational complexity

PODNO: Proper Orthogonal Decomposition Neural Operators

Zhongjian Wang(王中剑)
Nanyang Technological University

In this paper, we introduce Proper Orthogonal Decomposition Neural Operators (PODNO) for solving partial differential equations (PDEs) dominated by high-frequency components. Building on the structure of Fourier Neural Operators (FNO), PODNO replaces the Fourier transform with (inverse) orthonormal transforms derived from the Proper Orthogonal Decomposition (POD) method to construct the integral kernel. Due to the optimality of POD basis, the PODNO has potential to outperform FNO in both accuracy and computational efficiency for high-frequency problems. From analysis point of view, we established the universality of a generalization of PODNO, termed as Generalized Spectral Operator (GSO). In addition, we evaluate PODNO's performance numerically on dispersive equations such as the Nonlinear Schrödinger (NLS) equation and the Kadomtsev–Petviashvili (KP) equation.

Diffusion differentiable resampling

Zheng Zhao(赵正)
Linköping University

This works concerned with differentiable resampling in the context of sequential Monte Carlo (e.g., particle filtering).

We propose a new informative resampling method that is instantly pathwise differentiable, based on a diffusion bridge model.

We prove that our diffusion resampling method provides a consistent estimate to the resampling distribution, and we also empirically show that it outperforms the state-of-the-art differentiable resampling methods when used for stochastic filtering and parameter estimation.

Data and Model Jointly Driven Intelligent Nonlinear Filtering

Ji Shi(时骥)
Capital Normal University

In the era of big data, the demand for efficient and accurate data processing techniques has surged across various domains. This report introduces a novel intelligent filtering algorithm that is jointly driven by data and models, aiming to enhance the precision and efficiency of data filtering processes. The algorithm leverages the strengths of both data-driven and model-based approaches to achieve superior performance in diverse applications. The data-driven component of the algorithm utilizes large-scale datasets to identify patterns and regularities, ensuring that the filtering process is grounded in real-world observations. By employing advanced machine learning techniques, the algorithm can adapt to changing data distributions and improve its filtering accuracy over time. On the other hand, the model-driven component incorporates domain-specific knowledge and

predefined rules to guide the filtering process. This approach ensures that the algorithm adheres to established standards and constraints, enhancing its reliability and applicability in various contexts. By integrating model-based techniques, the algorithm can make informed decisions even in the absence of extensive data, thus broadening its scope of application. The algorithm achieves high accuracy, efficiency, and adaptability by leveraging the complementary strengths of data and model-driven approaches, making it a valuable tool for various data processing tasks.

Regularity Estimate of pathwise robust DMZ equation and its sparse approximation

Zhiwen Zhang(张智文)
University of Hong Kong

In this talk, we establish a priori estimates for arbitrary-order derivatives of solutions to the pathwise robust Duncan-Mortensen-Zakai (DMZ) equation within the framework of weighted Sobolev spaces. The weight function, which vanishes on the physical boundary, plays a crucial role in completing the a priori estimate proof. We employ Sobolev inequalities and their weighted counterparts to sharpen the regularity bounds, achieving improvements in both classical Sobolev spaces and Hölder continuity estimates. The refined regularity estimates reinforce the theoretical foundation of the low-rank QTT method and rigorously establish its convergence conditions. To enhance the method's capacity for solving high-dimensional forward Kolmogorov equations (FKEs) in real-time, we further refine the method for cases with functional polyadic drift f and observation h. Numerical experiments validate our theoretical results. For high-dimensional cubic sensor problems, our method demonstrates superior computational efficiency and accuracy compared to particle filter (PF) and extended Kalman filter (EKF) approaches.

A Comprehensive Survey of Nonlinear Filtering: Theory, Computation, and New Frontiers

Songlin Zhou(周松霖)
Tsinghua University

This report presents a comprehensive survey of nonlinear filtering problems, ranging from mathematical foundations to advanced numerical algorithms. We begin by deriving the fundamental stochastic differential equations governing the filtering process, specifically the Kolmogorov forward equations (KFEs) via the Girsanov theorem, as well as the Zakai and Kushner-Stratonovich stochastic equations. We then rigorously examine the solvability of these filtering equations by establishing a classification theorem for finite-dimensional estimating algebras. This theoretical framework provides the sufficient and necessary coefficient conditions for finite-dimensional Zakai equations and motivates the construction of the Yau-Yau filter. To address scenarios where these conditions are not met, we introduce the Yau-Yau algorithm, a numerical method that efficiently solves KFEs by leveraging an offline-online computational decomposition. Finally, we review recent advancements in the Yau-Yau algorithm, particularly its integration with neural networks for solving high-dimensional filtering problems.