Residues of differential forms on singular varieties

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For a smooth manifold M and the space $\Lambda^p(M)$ of all differential p -forms on M the restriction $\omega|_N$ of $\omega \in \Lambda^p(M)$ to a smooth submanifold $N \subset M$ is determined by the geometry of N. If N is any subset of M then the forms $\alpha + d\beta, \alpha \in \Lambda^p(M), \beta \in \Lambda^{p-1}(M)$, where α and β annihilates any p -tuple (and $p-1$ - tuple respectively) of vectors in $T_xM, x \in N$, are called algebraically vanishing on N or having zero algebraic restriction to N. An algebraic restriction of $\omega \in \Lambda^p(M)$ to N is defined as an equivalence class of ω modulo forms with zero algebraic restriction to N. We study germs of differential forms over singular varieties. The geometric restriction of differential forms to singular varieties is introduced and algebraic restrictions of differential forms with vanishing geometric restrictions, called residual forms, are investigated. Residues of plane curves-germs, hypersurfaces, Lagrangian varieties as well as the geometric and algebraic restriction via a mapping were examined.

Bernstein-Sato roots and test module filtrations for Cartier pairs in positive characteristics

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The Bernstein-Sato polynomial is an important invariant of an ideal J in a polynomial ring $\mathcal{C}[x_1, \ldots, x_n]$, which measures the singularities of $\mathbf{V}(J)$, the zero-locus of J, in very subtle ways. The roots of this polynomial are closely related to the jumping numbers of the multiplier ideals of the pair $(\mathcal{C}^n, \mathbf{V}(J))$.

Work of M. Mustata's, later extended by T. Bitoun and E. Quinlan-Gallego, provides an analogous Bernstein-Sato theory for an ideal I in a regular F-finite ring R of positive characteristic. M. Blickle and A. Stäbler generalized M. Mustata's constructions and defined a family of Bernstein-Sato polynomials for a F-regular Cartier module associated with a principal ideal.

Now let $I = \langle f_1, \ldots, f_r \rangle$ be an ideal of a regular F-finite ring R of characteristic $p > 0$, and let $(M, \mathcal{C})$ be an R-Cartier pair. Based on the graph embedding along I and the multi-eigen-decomposition, we define the Bernstein-Sato roots for the triple $(M, \mathcal{C}, I)$ under some certain conditions. I will show that these roots have similar properties as in the case I is principal or $M = R$. They are non-positive, rational, and closely related to the F-jumping numbers of the non-increasing right-continuous filtration, the test module filtration $\{\tau(M, \mathcal{C}^{I(t)})\}_{t \geq 0}$, which is an analogue of V-filtration in the complex case.

On the monodromy conjecture for determinantal varieties

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This paper presents a proof of the monodromy conjecture for determinantal varieties. Our strategy centers on an in-depth analysis of monodromy zeta functions, leveraging a generalized A'Campo formula, an examination of multiple contact loci, and the exploitation of the intrinsic symmetric structures inherent to these varieties. Furthermore, we prove the holomorphy conjecture for determinantal varieties and the monodromy conjecture for Brill-Noether loci of generic curves.

Bounded Negativity Conjecture for Fermat surfaces

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The Bounded Negativity Conjecture predicts that the self-intersection numbers of reduced and irreducible curves on a given smooth projective surface are bounded from below by a constant that depends only on the surface. This long-standing conjecture has remained open for a century. In this talk, I will investigate the conjecture for Fermat surfaces by giving an explicit formula for all negative curves, expressed in terms of the degree of the surface and local numerical invariants for curve singularities. The result is expected to provide new insights in the study of the Bounded Negativity Conjecture.

Removability of isolated singularities of critical points of parametric elliptic functionals

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It is well-known that for holomorphic functions (therefore, also for harmonic functions), there are three types of isolated singularities: removable singularities, poles, and essential singularities. Although minimal surfaces are locally solutions of a second order elliptic partial differential equation like harmonic functions, isolated singularities of minimal surfaces are all removable (Bers 1951). Bildhauer-Fuchs (2022) generalized Bers' result to critical points of axially-symmetric parametric elliptic functionals: a natural generalization of surface area. In this talk, we discuss removability of isolated singularities of critical points of more general parametric elliptic (and non-elliptic) functionals.

Quadratic Maps: geometry of their critical value set and focal set, and their polar dual relation

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We take a quadratic map $Q : R^k --> R^n$ and consider its restriction f to the unit sphere in R^k. We prove that the projective dual variety of the critical value set of f is an algebraic hypersurface of degree k, noted $F(f)$, which is hyperbolic: every line through the origin of R^n intersects $F(f)$ at k real points (counting multipmlicities). This projective duality and the hyperbolicity of $F(f)$ provide topologic and geometric properties of the critical value set of f and of its focal set. This has direct applications to local differential geometry of k-dimensional submanifolds in Euclidean space $R^N(N = K + n)$.

Singularities and Limits of F-invariants

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For Noetherian local rings of positive characteristic, the Hilbert-Kunz multiplicity and F-signature are fundamental invariants measuring different types of singularities. These singularities are related to singularities in characteristic 0 via reduction mod p. However, many properties of both invariants remain unknown.

In this talk, we prove an inequality conjectured by Watanabe and Yoshida, which states that for every odd prime p, the Hilbert-Kunz multiplicity of the Fermat hypersurface of degree 2 in characteristic p is greater than or equal to its limit when p goes to infinity. This conjecture describes the possible lower bound of Hilbert-Kunz multiplicities of singular rings in fixed characteristic and dimension. The proof of this conjecture uses representation theory of certain $k[T]$-modules and mathematical analysis.

Higher order Hessian matrix theory and its applications in Calabi-Yau manifolds

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One of the fundamental problems in algebraic geometry and singularity theory is to investigate whether two given smooth projective manifolds X and Y are projective equivalent. In hypersurface case, when their defining equations have degree 2, this can be resolved using quadratic form theory and classical Hessian matrix theory. For cases with degrees greater than 2, in this talk, we shall develop the novel "higher order Hessian matrix theory" as a generalization of classical Hessian matrix theory. Many new invariants for projective manifolds (especially Calabi-Yau manifolds) are obtained beyond the classical theory. With this sequence of invariants, we solve a several decades old problem about the classification of complex structures of K_3 surfaces in $\mathbb{CP}^3$.

Canonical blowups of Grassmannians and the Faltings-Lafforgue compactifications

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We give a linear algebraic construction of the Lafforgue spaces associated to the Grassmannians Gr(2,n) by blowing up certain explicitly defined monomial ideals, which sharpens and generalizes a result of Faltings. In this talk, I will briefly recall results of Faltings and Lafforgue's work respectively, and show how we extend their construction to the case for arbitrary diagonal subtorus actions beyond the equidimensional ones. This is a joint wok with Hanlong Fang. See https://arxiv.org/abs/2310.17367 .

Polar loci of multivariable archimedean zeta functions

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The study of archimedean zeta functions started from a problem proposed by Gel'fand at ICM 1954, about the meromorphy of the distribution f^s. In 1980s, Barlet related the poles of archimedean zeta functions with monodromy eigenvalues, endowing these poles with a geometrical explanation. In this talk, I will present a generalization of Barlet's result to multivariable cases. As an application, a full characterization of the oblique slopes of generalized Bernstein-Sato ideals will be given. This is a joint work with Nero Budur and Huaiqing Zuo.

Discontinuous gradient ODEs, trajectories in the minimal action problem, and massive points in one cosmological model

Ilya Bogaevskii
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The gradient of a concave function is discontinuous vector field but has well-defined trajectories. We formulate an existence and semi-uniqueness theorem and its generalisation for non-stationary case. Using the latter we construct trajectories in the minimal action problem and investigate how massive points appear. Their formation simulates the large-scale matter distribution in one of the simplest cosmological models based on the Burgers equation.

Tjurina Number Jumps and Unimodal Hypersurface Singularities in Positive Characteristic

Hongrui Ma(马泓锐)
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In 2016, Greuel and Nguyen extended the concept of modality of hypersurface singularities to arbitrary algebraically closed fields and classified hypersurface singularities of modality 0 in positive characteristic under right equivalence. We generalize their method to obtain sharper bounds on modality and complete the classification of unimodal hypersurface singularities under contact equivalence. We find that each sudden jump in the Tjurina number necessarily increases the modality.

Global singularity theory and its application to data analysis

Osamu Saeki
Kyushu University, Japan

In this talk, we first consider smooth generic maps $f : MN$ between smooth manifolds M and N with $dimM \geq dimN \geq 1$ and explore various topological properties of their singular fibers. A singular fiber is, roughly speaking, the inverse image f^{-1}(y) for a singular value $y \in N$ of f. We explain how the singular fibers are classified under various settings. Such classification results lead to various interesting consequences, such as classification of local topological structures of Reeb spaces, or cobordism invariants for maps. We also have some applications to data visualizations, especially for developing new technologies for visualizing large scale data. If time permits, we will mention the possibility to use them to design topological loss functions.

Classifying open subsets with the same quotient

Xucheng Zhang(张旭成)
Beijing University of Aeronautics and Astronautic, China

For a smooth Gm-variety X that admits a good quotient, we classify all open subsets with the same good quotient as X, whose existence depends on the Białynicki-Birula decomposition of X. This talk is based on a joint work in progress with Jianping Wang (BIMSA).

Local differential geometry of lightcone framed submanifolds

Liang Chen(陈亮)
Northeast Normal University, China

We construct a foundational research framework for the local differential geometry of lightcone-

framed submanifolds in Lorentzian 3-manifolds, which include both curves and surfaces.

Immersions associated with links of complex surface singularities and their topology

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In this talk, we study links of isolated surface singularities in $\mathbb{C}^3$ from the viewpoint of immersions through two problems. First, for each singularity, we determine the regular homotopy class of the inclusion map of its link into the 5-sphere. Second, for each type of Arnol'd's simple singularities, we show that the inclusion map of its link is regularly homotopic to the immersion associated with the corresponding Dynkin diagram which was constructed by Kinjo. We prove these by computing the complete invariants of the immersions given by Wu and Saeki--Szűcs--Takase. As an application, we also determine the Smale invariants of Kinjo's immersions.

Discrete Morse Theory as a Framework for Discretizing Classical Singularities

Chong Zheng(郑翀)
Waseda University, Japan

In this talk, we introduce our recent developments in discretizing key ideas from classical singularity theory using discrete Morse theory, introduced by Robin Forman in the 1990s. By establishing a connectedness theory for discrete gradient vector fields that closely mirrors the smooth case, we can describe the birth and death of critical points and thereby construct a discrete counterpart of Cerf theory. These constructions demonstrate that discrete Morse theory is not merely a tool for simplifying complexes, but a setting in which singular behavior itself can be represented and analyzed.

Building on these previous results, and in collaboration with Prof. Toru Ohmoto, we will sketch a broader vision. We aim to apply discrete Morse structure as a foundation for interpreting invariants arising from smooth or stratified geometry, including characteristic classes and intersection homology. This perspective suggests a path toward extending ideas from singularity theory into non-smooth contexts. The goal is to propose a coherent framework in which classical singularity theory and combinatorial topology mutually enrich one another. This opens new theoretical directions as well as potential interactions with computational fields such as topological data analysis and machine learning.

Brain, sex, temperature and memory performance

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Understanding gender differences in thinking and memory can reveal important details about how the human brain works. In our study, we used advanced tools — magnetic resonance thermometry, functional MRI (fMRI), and standard working memory tests (n-back tasks) — to see whether differences in brain temperature between men and women relate to how their brains function during memory tasks.

We found that men's average brain temperature dropped during memory activity, while women's brain temperature remained more stable. Although temperature changes were smaller in women, we discovered an inverse relationship between how much their brain temperature changed and how well they performed. These changes were also linked to differences in BOLD (blood oxygen level–dependent) signals, which indicate neural activity.

In women, this connection between temperature, performance, and brain activity was strong, but it was not observed in men. Additionally, women showed extra brain activation patterns while achieving the same performance levels as men.

This suggests that women may compensate for greater temperature sensitivity by engaging additional brain regions to maintain strong working memory performance.

Overall, our findings highlight that gender differences in brain function are complex and that temperature changes play a meaningful role in shaping how the brain supports cognitive tasks. This opens new directions for understanding how thermal regulation affects thinking and mental performance.

L2 estimates for line bundles equipped with singular Hermitian metrics

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Beijing University of Posts and Telecommunications, China

The L2 estimates concerning solutions of the $\bar{\partial}$ equations which are important tools in finding holomorphic functions that satisfies various prescribed conditions. In many cases, the prescribed condition can be characterized by the singularities of singular Hermitian metrics(whose weight are plurisubharmonic functions). Multiplier ideal sheaves have become an indispensable tool for analyzing the singularities of singular metrics and are also essential for algebraic geometric applications. In this talk, I will give some vanishing results for cohomology groups in terms of Multiplier ideal sheaves.

log transversality, global Ginzburg's sharp construction and applications

Xia Liao(廖侠)
Huaqiao University, China

We discuss log characteristic cycles of constructible functions along a Saito free divisor D and global Ginzburg's sharp construction. When characteristic cycles are log transverse to D, certain global index formulas hold. we discuss two applications of log transversality and global index formulas. 1. a characterization of strong Euler homogeneity of projective hypersurfaces using syzygies of Jacobian ideals. 2. local and global index formulas for GSV indices of 1-dimensional holomorphic foliations. This talk is based on my joint work with Xiping Zhang.

Algebraic Geometric Construction on Superintegrable Systems in a Homogeneous Space

Guorui Ma
Tsinghua University, China

In this work, we have presented an approach for constructing (super)integrable geodesic flows on a class of symmetric symplectic manifolds using algebraic and geometric methods. In particular, using invariant polynomial functions in Lie algebras and their Poisson-commutative properties, we obtained explicit first integrals for magnetic geodesic flows in $T^*(G/A)$, where G/A is an adjoint orbit. An important observation from our work is the algebro-geometric construction of the integrals of motion, which provides a unifying perspective on various classical approaches (such as the argument shift method and the Gelfand-Cetlin-type constructions) within a single coherent framework in the language of Poisson projection chains. We also want to build a similar framework on an orbifold or a stratifold (for example when the action is not free) which will be extramely singular. This is a joint work with Kai Jiang, Ian Marquette, Junze Zhang and Yao-Zhong Zhang.