



DYNAMICS, TEICHMULLER THEORY
AND THEIR RELATED TOPICS
动力系统、泰希米勒理论
及其相关主题研讨会



MARCH 31ST TO APRIL 4TH, 2023

# DYNAMICS. TEICHMULLER THEORY AND THEIR RELATED TOPICS 动力系统、泰希米勒理论及其相关主题研讨会

# Schedule

# 2023"动力系统,Teichmuller理论及相关领域"学术会议 日程安排

2023年3月31日至4月4日

时间	3月31日 (周五)	4月1日 (周六)	4月2日 (周日)	4月3日 (周一)	4月4日 (周二)
9:00-9:45	刘毅	王跃飞	谢俊逸	范爱华	张翼华
9:55-10:40	万学远	苏伟旭	孙哲	郭常予	李智强
10:45-11:05	茶歇	合影及茶歇	茶歇		
11:05-11:50	薛宇皓	周泽	胡飞	聂鑫	冀诸超
12:00-13:30	午餐				
14:00-14:45	戎锋	杨飞		杨文元	张影
14:55-15:40	高延	洪寒		潘会平	陈绍示
15:40-16:10	茶歇		交流讨论	茶歇	
16:10-16:55	高悦	曹杰		沈洋	交流讨论
17:05-17:50	杨舍			任浩杰	
18:00	晩餐		晚餐	晚餐	





## Title and abstract

#### 1.Yi Liu, Peking University

Title: Mapping tori and profinite properties

**Abstract:** In this talk, I will discuss recent results in studying profinite properties of hyperbolic 3-manifolds, and explain how they are connected to Nielsen fixed point theory of surface automorphisms.

#### 2.Xueyuan Wan, Chongqing University of Technology

**Abstract:** This talk deals with the representations of the fundamental groups of compact surfaces with boundary into classical simple Lie groups of Hermitian type. We relate work on the signature of the associated local systems of Atiyah-Patodi-Singer, to Burger-Iozzi-Wienhard's Toledo invariant. To measure the difference, we extend Atiyah-Patodi-Singer's rho invariant, initially defined on U(p), to discontinuous class functions, first on U(p,q), and then on other classical groups via embeddings into U(p,q). In this way, we present three different invariants – signature, Toledo and rho invariant – in a unifying way, which is a version of the classical signature formula of

Atiyah-Patodi-Singer for manifolds with boundary. This work is joint with Prof. Inkang Kim and Prof. Pierre Pansu.

#### 3. Yuhao Xue, Tsinghua University

**Title:** Separating systole for random hyperbolic surfaces of Weil-Petersson model

**Abstract:** The Weil-Petersson metric on the moduli space of genus g hyperbolic surfaces is of finite volume, and hence induce a probability measure. In this talk, we will discuss the behavior of some geometric quantities, especially the separating systole, for random hyperbolic surfaces with respect to this Weil-Petersson measure. We show that the length of separating systole is approximately  $2\log(g)$ - $4\log(\log(g))$  and it separates out a one-holed torus for random hyperbolic surfaces. This talk is based on joint works with Xin Nie, Hugo Parlier and Yunhui Wu.

#### 4. Feng Rong, Shanghai Jiao Tong University

Title: On the local dynamics in Fuchsian directions

**Abstract:** In the first part of the talk, we give a brief survey of the local holomorphic dynamics in higher dimensions, focusing on maps tangent to the identity. In the second part of the talk, we present some new sufficient conditions for the existence of attracting domains in Fuchsian characteristic directions.

#### 5. Yan Gao, Shenzhen University

**Title:** Invariant graph and its application in complex dynamics

**Abstract:** Invariant graph is a fundamental tool in complex dynamics, since it induces a Markov partition on the Julia set. In the polynomial case, there are two kinds of natural invariant graphs, namely Yoccoz puzzle and Hubbard tree. But it is still an open problem to construct invariant graphs for general rational maps. In the talk, I will introduce the progress on this topic, including my joint works with G. Cui, H. Nie, J. Zeng.

#### 6. Yue Gao, Anhui Normal University

**Title:** Shape of Thurston's filling systole subset in surface moduli space **Abstract:** In this talk, I am going to talk about the sparseness of Thurston's subset. Sparseness is a geometric concept on Thurston's subset firstly raised by Anderson-Parlier-Pettet in 2016. We have proved the sparseness of Thurston's subset in the sense of Teichmüller distance and Weil-Petersson distance. More precisely, most surfaces in genus g surface moduli space have Teichmüller distance \$\frac{1}{5}\log\log g\$ and Weil-Petersson distance \$0.6521(\sqrt{\log g}-\sqrt{7\log\log g})\$ to the Thurston's subset. Some recent progresses on random surface (Mirzakhani-Petri, Nie-Wu-Xue) and estimate of the Weil-Petersson distance by systole (Wu) are important tools in our proof.

#### 7. She Yang, Peking University

**Title:** Dynamical Mordell-Lang conjecture for totally inseparable liftings of Frobenius

Abstract: We prove that if K is a complete algebraically closed non-archimedian valuation field of positive characteristic and f is an endomorphism of the projective space over K which is totally inseparable and behaves as the Frobenius on the special fiber, then f satisfies the dynamical Mordell-Lang (DML) property.



#### 8. Yuefei Wang, Shenzhen University

Title: On Collatz Conjecture

**Abstract:** We will talk about recent results of the Collatz conjecture about the periodic oibits of the Collatz maps on  ${\mathbb Z}_2$ .

#### 9.Weixu Su, Fudan University

**Title:** The geometry of horospheres in Teichmuller space and its applications

Abstract: The study of the Teichmuller metric is inspired by analogies with geodesic spaces of negative curvature. In this talk, we will discuss some geometric properties of the horospheres in Teichmuller space. As applications, we obtain: (1) A new proof of the Royden Theorem that every isometry of the Teichmuller metric is induced by an element of the mapping class group. (2) A sufficient and necessary condition when two points in the horofunction boundary of Teichmuller space are "connected" by a Teichmuller geodesic. The work is jointed with Dong Tan and Xiaoke Lou.

#### 10.Ze Zhou, Shenzhen University

**Title:** Polygons inscribed in Jordan curves with prescribed edge ratios **Abstract:** Let J be a smooth Jordan curve. For a tuple of positive numbers  $a_1, \dots, a_n$ , each of which is less than the sum of the others, we show that there exists a polygon  $Q_n$  inscribed in J with sides of lengths  $\mu a_1, \dots, \mu a_n$  for some .  $\mu > 0$ . This is a joint work with Yaping Xu.

#### 11. Fei Yang, Nanjing University

**Title:** Rational maps with smooth degenerate Herman rings

Abstract: We prove the existence of rational maps having smooth degenerate Herman rings. This answers a question of Eremenko affirmatively. The proof is based on the construction of smooth Siegel disks by Avila, Buff and Chéritat as well as the classical Siegel-to-Herman quasiconformal surgery. A crucial ingredient in the proof is the surgery's continuity, which relies on the control of the loss of the area of quadratic filled-in Julia sets by Buff and Chéritat. As a by-product, we prove the existence of rational maps having a nowhere dense Julia set of positive area for which these maps have no irrationally indifferent periodic points, no Herman rings, and are not renormalizable.

#### 12.Han Hong, Tsinghua University

Title: Large Steklov eigenvalue on hyperbolic surfaces

Abstract: The first nonzero Steklov eigenvalue on a Riemannian surface is controlled by the surface's topology. As a result, a sequence of surfaces with a divergent first nonzero Steklov eigenvalue will have its topology blowing up. We presented new constructions for this sequence of surfaces with divergent eigenvalues but a fixed number of boundary components. Our work is inspired by the excellent and fundamental contributions of Mirzakhani and Nie-Wu-Xue in the theory of random surfaces. Additionally, we explored hyperbolic surfaces with a connected boundary and large Steklov eigenvalue in the Moduli space. This is a joint work with X. L. Han and Y. X. He.

#### 13.Jie Cao, Zhejiang University

**Title:** Boundaries of capture hyperbolic components

**Abstract:** In complex dynamics, the boundaries of high dimensional hyperbolic components in a reasonable space of polynomials or rational maps are mysterious objects. In this talk, we discuss some typical families of polynomials defined by periodic critical relations, and show that the boundary of a capture hyperbolic component is homeomorphic to the sphere. Moreover, we establish a relation for Hausdorff dimensions between the parameter space and the dynamical space. This is a joint work with Xiaoguang Wang and Yongcheng Yin.

#### 14.Junyi Xie, Peking University

Title: Rigidity of length spectrum in complex dynamics

**Abstract:** This talk is based on a joint work with Zhuchao Ji. We show that aside from the flexible Lattès family, the length spectrum of periodic points determines the conjugacy class of rational maps up to finitely many choices. This generalize the McMullen's rigidity theorem for multiplier spectrum.

#### 15.Zhe Sun, University of Science and Technology of China

Title: Asymmetric intersection number and Fock-Goncharov duality

**Abstract:** Fock and Goncharov introduced a pair of mirror moduli spaces associated to G and G^L which generalized the Teichmüller space and the decorated Teichmüller space, and they proposed a duality: the canonical



basis of the regular function ring of one space X is parameterized by the tropical integral points of its mirror  $X^V$ . In this talk, I will explain my joint work with Linhui Shen and Daping Weng, where we introduce the asymmetric intersection numbers between webs on the surface to investigate the duality for SL3 (in progress for Sln).

#### 16.Fei Hu, Nanjing University

**Title:** An upper bound for polynomial volume growth of automorphisms of zero entropy

**Abstract:** Let X be a smooth projective variety (or a compact Kähler manifold) of dimension  $d \ge 2$  and f an automorphism of X.

Suppose that the pullback  $f^*|_{H^{1,1}(X)}$  of f on the Dolbeault cohomology  $H^{1,1}(X)$  is unipotent and denote the index of the eigenvalue by k+1.

We prove an upper bound for the polynomial volume growth plov (f) of f, or equivalently, for the Gelfand--Kirillov dimension of the twisted homogeneous coordinate ring associated with (X, f) as follows:  $plov(f) \le (k/2 + 1)d$ .

Combining with the inequality  $k \le 2(d-1)$  due to Dinh--Lin--Oguiso--Zhang, we obtain an optimal inequality that  $plov(f) \le d^2$ ,

which affirmatively answers questions of Cantat--Paris-Romaskevich and Lin--Oguiso--Zhang.

This is joint work with Chen Jiang.

#### 17. Aihua Fan, Central China Normal University

Title: Weighted Birkhoff averages and Bohr Chaoticity

Abstract: The Sarnak conjecture, which concerns with the Birkhoff averages weighted by the Möbius sequence, asserts that all zero entropy systems are orthogonal to the Möbius sequence. Which systems are orthogonal to none of non-trivial weights? We define such systems as Bohr chaotic systems. The Bohr chaoticity is a complexity measure and is a topological invariant; it implies the positivity of entropy. However, the positivity of entropy doesn't imply the Bohr chaoticity. We prove that a system (X, T) admitting a horseshoe (i.e a susbsytem of some power T is conjugate to a full shift) is Bohr chaotic. Thus the usual nice systems of positive entropy are Bohr chaotic. But systems having few ergodic measures are not Bohr chaotic. Another class of systems which are proved to be Bohr chaotic are the algebraic principal systems.

The presented results are joint works with Shilei FAN (Wuhan), Valery RYZHYKOV (Moscou), Klaus SCHMIDT (Vienna), Weixiao SHEN (Shanghai) et Evgeny VERBITSKIY (Leiden).

#### 18.Changyu Guo, Shandong University

Title: Geometric parametrization of metric spaces

**Abstract:** In this talk, we shall give an overview of the geometric parametrization problem of metric spaces, which is a very active topic in the recent research about quasiconformal analysis. We shall also discuss some natural connecitons with Analysis on metric spaces, Geometric measure theory, Geometric analysis (around harmonic maps) and Harmonic analysis.

#### 19.Xin Nie, Southeast University

Title: Higher Teichmüller theory and minimal surfaces in pseudohyperbolic spaces

**Abstract:** We will first explain 1) the link between Teichmüller theory and maximal surfaces in the anti-de Sitter space (works of Mess and Bonsante-Schlenker); 2) Higgs bundles and surface group representations in Lie groups ("higher Teichmüller theory"); 3) a generalization of 1) to higher Teichmüller theory, which solves a conjecture of Labourie for the Lie group SO(2,n) (work of Collier-Tholozan-Toulisse). Then we will talk about a further generalization of 3) which allows us to tackle the exceptional Lie group  $G_2$ .

#### 20.Wenyuan Yang, Peking University

**Title:** Conformal density for groups with contracting elements

Abstract: In 1976, S. Patterson introduced a class of conformal measures on the limit set of Fuchsian groups, and further developed by D. Sullivan in Kleinian groups with various applications in spectral geometry, Mostow rigidity, geodesic flows and complex dynamics etc. These now called Patterson-Sullivan measures are important instances of conformal density for rank-1 symmetric spaces. In this talk, we will explain how to establish a theory of Patterson-Sullivan measures on the horofunction boundary for any discrete group on a geodesic metric space with contracting elements. Such group actions include CAT(0) groups and mapping class groups, so our theory could be applied. Applications to growth problems and geodesic flows shall be mentioned, if time permits.

#### 21.Huiping Pan,South China University of Technology

Title: Ray structures on the Teichmuller space

Abstract: Teichmuller space admits several ray structures, such as the



Teichmuller geodesic ray, Thurston stretch ray, harmonic map (dual) ray, grafting ray, etc. In the first part of this talk, we will depict harmonic map ray structures on Teichmuller space as a geometric transition between Teichmuller ray structures and Thurston geodesic ray structures. In particular, by appropriately degenerating the source of a harmonic map between hyperbolic surfaces, the harmonic map rays through the target converge to a Thurston geodesic; by appropriately degenerating the target of the harmonic map, those harmonic map dual rays through the domain converge to Teichmuller geodesics. In the second part, we will discuss applications to Thurston geodesics. While there may be many Thurston metric geodesics between a pair of points in Teichmuller space, we find that by imposing an additional energy minimization constraint on the geodesics, thought of as limits of harmonic map rays, we select a unique Thurston geodesic through those points. Extending the target surface to the Thurston boundary yields, for each point Y in Teichmuller space, an "exponential map" of rays from that point Y onto Teichmuller space with visual boundary the Thurston boundary of Teichmuller space. This is a joint work with Michael Wolf.

#### 22. Yang Shen, Tsinghua University

**Title:** On the Cheeger constants of Belyi surfaces

**Abstract:** Brooks and Makover developed a combinatorial model of random hyperbolic surfaces by gluing certain hyperbolic ideal triangles. In this talk we show that for any \$\epsilon>0\$, as the number of ideal triangles goes to infinity, a generic hyperbolic surface in Brooks-Makover's model has Cheeger constant less than \$\frac{3}{2\pi}+\epsilon\$. This is a joint work with Yunhui Wu.

#### 23.Haojie Ren,Fudan University

Title: A Dichotomy for the dimension of SRB measure

Abstract: We study dynamical systems generated by skew products:

 $T:[0,1)\times R \rightarrow [0,1)\times R$ 

 $T(x,y) = (bx \mod 1, y+\phi(x))$ 

where integer, and is a real analytic Z-periodic function. We prove the following dichotomy for the SRB measure for T: Either the support of is a graph of real analytic function, or the dimension of is equal to. Furthermore, given b and, the former alternative only happens for finitely many unless is constant.

#### 24. Yitwah Zhang, Tsinghua University

Title: Geodesics in moduli space

Abstract: There are many canonical metrics on the moduli space of closed Riemann surfaces, of which the Teichmuller metric is the best understood. Not only is it known that the typical geodesics of the Teichmuller metric is dense, but further it is known that relatively compact ones form a set of measure zero, albeit of full Hausdorff dimension. Many of these results about Teichmuller geodesics are inspired by corresponding results regarding symmetric spaces and exploit analogies from Diophantine approximation. In this talk, I will give a survey of these results about Teichmuller geodesics and some open problems inspired by them.

#### 25.Zhiqiang Li, Peking University

Title: Recent progresses in ergodic optimization

**Abstract:** In this talk, we discuss some recent progresses in an area in ergodic theory called ergodic optimization. The focus of ergodic optimization is on the properties of invariant measures that maximizes the integral of potential functions, which has close connections to zero temperature behaviors of dynamical systems and multifractal analysis.

#### 26.Zhuchao Ji,Westlake University

Title: Dynamical André-Oort conjecture for curves

**Abstract:** This is a joint work with Junyi Xie. The Dynamical André-Oort conjecture for curves proposed by Baker and DeMarco asserts the following: Given an non-isotrivial algebraic family of rational maps parametrized by an algebraic curve, then there are infinitly many PCF parameters in this curve if and only if the family has at most one independent critical orbit. In this talk we will sketch a proof of this conjecture.

#### 27. Ying Zhang, Soochow University

**Title:** Trace polynomials for closed curves on the enlarged modular torus: Monotonicity, positivity, and log-concavity

**Abstract:** The modular torus is a once-cusped hyperbolic torus with the maximal order of symmetry. The traces of the simple closed geodesics on the modular torus give the geometric version of the classical Markoff numbers. We study trace polynomials for the closed curves on the enlarged



modular torus, with the single variable the enlargement factor. We obtain partial ordering of the polynomials for certain simple closed curves, and positivity of the polynomials for all closed curves. We have observed log-concavity for these polynomials and confirmed log-concavity of the polynomials for the simple closed curves. We mention the problem to enlarge an arbitrary complete one-holed hyperbolic torus. This is joint work with Xiangfei Li.

#### 28. Shaoshi Chen, Academy of Mathematics and Systems Science, Chinese Academy of Sciences

Title: Dynamics of P-recursive Sequences

**Abstract:** P-recursive sequences satisfy a linear recurrence with polynomial coefficients, which appear ubiquitously in combinatorics. This talk will present some dynamical results on P-recursive sequences related to the stability problem and the Skolem problem on the zero structure of these sequences.

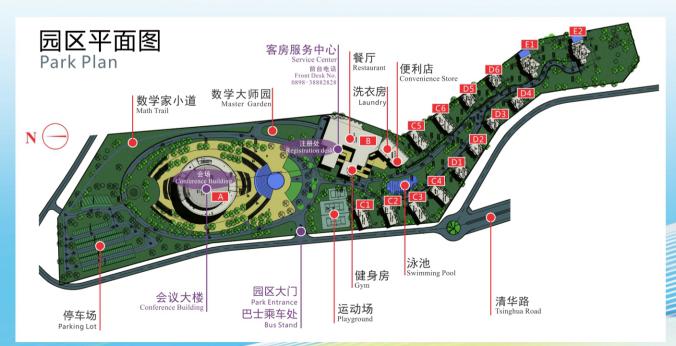
## **Welcome to TSIMF**



The facilities of TSIMF are built on a 23-acre land surrounded by pristine environment at Phoenix Hill of Phoenix Township. The total square footage of all the facilities is over 29,000 square meter that includes state-of-theart conference facilities (over 10,000 square meter) to hold many international workshops

simultaneously, two libraries, a guest house (over 10,000 square meter) and the associated catering facilities, a large swimming pool, workout gym and sport courts and other recreational facilities.

Mathematical Sciences Center (MSC) of Tsinghua University, assisted by TSIMF's International Advisory Committee and Scientific Committee, will take charge of the academic and administrative operation of TSIMF. The mission of TSIMF is to become a base for scientific innovations, and for nurturing of innovative human resource; through the interaction between leading mathematicians and core research groups in pure mathematics, applied mathematics, statistics, theoretical physics, applied physics, theoretical biology and other related fields, TSIMF will provide a platform for exploring new directions, developing new methods, nurturing mathematical talents, and working to raise the level of mathematical research in China.





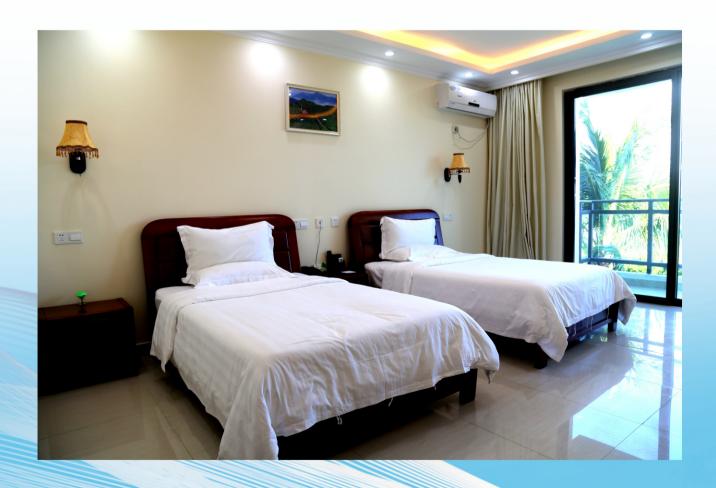
## Registration

Conference booklets, room keys and name badges for all participants will be distributed at the front desk. Please take good care of your name badge. It is also your meal card and entrance ticket for all events.

## **Guest Room**

All the rooms are equipped with: free Wi-Fi (no password), TV, air conditioning and other utilities.

Family rooms are also equipped with kitchen and refrigerator.



## Library

Opening Hours: 09:00am-22:00pm

TSIMF library is available during the conference and can be accessed by using your room card. There is no need to sign out books but we ask that you kindly return any borrowed books to the book cart in library before your departure.





In order to give readers a better understanding of the contributions made by the Fields Medalists, the library of Tsinghua Sanya International Mathematics Forum (TSIMF) instituted the Special Collection of Fields Medalists as permanent collection of the library to serve the mathematical researchers and readers.

So far, there are 234 books from 47 authors in the Special Collection of Fields Medalists of TSIMF library. They are on display in room A220. The participants are welcome to visit.



## Restaurant

All the meals are provided in the restaurant (Building B1) according to the time schedule.





**Breakfast** 07:30-08:30

**Lunch** 12:00-13:30

**Dinner** 17:30-19:00

# **Laundry**

Opening Hours: 24 hours The self-service laundry room is located in the Building 1 (B1).



# Gym

The gym is located in the Building 1 (B1), opposite to the reception hall. The gym provides various fitness equipment, as well as pool tables, tennis tables etc.

# **Playground**



Playground is located on the east of the central gate. There you can play basketball, tennis and badminton. Meanwhile, you can borrow table tennis, basketball, tennis balls and badminton at the reception desk.

# **Swimming Pool**





Please note that there are no lifeguards. We will not be responsible for any accidents or injuries. In case of any injury or any other emergency, please call the reception hall at +86-898-38882828.



## **Outside Shuttle Service**

We have shuttle bus to take participants to the airport for your departure service. Also, we would provide transportation at the Haihong Square (海虹广场) of Howard Johnson for the participants who will stay outside TSIMF. If you have any questions about transportation arrangement, please feel free to contact Ms. Li Ye (叶莉) at (0086)139-7679-8300.

# Free Shuttle Bus Service at TSIMF

We provide free shuttle bus for participants and you are always welcome to take our shuttle bus, all you need to do is wave your hands to stop the bus.



Destinations: Conference Building, Reception Room, Restaurant, Swimming Pool, Hotel etc.



# **Contact Information of Administration Staff**

Location of Conference Affairs Office: Room 104, Building A

Tel: 0086-898-38263896

Conference Manager: Shouxi He 何守喜

Tel: 0086-186-8980-2225 Email: hesx@tsimf.cn

Location of Accommodation Affairs

Office: Room 200, Building B1 Tel: 0086-898-38882828

Accommodation Manager: Ms. Li YE 叶莉

Tel: 0086-139-7679-8300 Email: yeli@tsimf.cn Director Assistant of TSIMF

Kai CUI 崔凯

Tel/Wechat: 0086- 136-1120-7077

Email:cuikai@tsimf.cn

Director of TSIMF Prof.Xuan GAO 高瑄 Tel: 0086-186-0893-0631

Email:gaoxuan@tsinghua.edu.cn

