

## **$p$ -isogenies with $G$ -structure**

**Si Ying Lee**

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I will talk about defining the notion of  $p$ -isogenies using the theory of  $F$ -gauges, and how this allows us to construct integral models of Hecke correspondences. I will also discuss some expected consequences of this, such as constructions of Rapoport-Zink spaces, and a general framework on understanding integral Hecke actions on Shimura varieties. This is joint work in progress with Keerthi Madapusi.

## **Finite Langlands correspondence**

**Naoki Imai**

University of Tokyo, Japan

In this talk, we discuss the formulation and construction of the Langlands correspondence for reductive groups over finite fields, which we call the finite Langlands correspondence. We discuss also its conjectural relation with the categorical local Langlands correspondence. This is partially based on a joint work with David Vogan.

## **Nearby cycles in the semistable case via gluing of stratified Tate motives**

**Hiroki Kato**

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The nearby cycle complex is relatively well-understood in the semistable case. For instance, in the Betti, étale, and motivic contexts, it is known to be an extension of constant objects (up to shift and twist) on the natural strata, respectively, by Steenbrink, Rapoport-Zink/Saito, Ayoub. Moreover, in the Betti and étale contexts, the extension class (i.e., the gluing datum) is analyzed well enough to fully understand the  $E_1$ -page of the weight spectral sequence. However, in any of the above contexts, a complete and concrete identification of the extension class does not seem to have been established. I will present an attempt towards such an identification.

## **Affine Grassmannians and close fields**

**Kazuhiro Ito**

Tohoku University, Japan

Let  $G$  be a split reductive group over  $\mathbb{Z}$  and  $\mu$  a cocharacter of  $G$ . For a local field  $F$  of characteristic  $p$  with finite residue field  $k$ , we have the Schubert variety over  $k$  corresponding to  $\mu$  in the affine Grassmannian of a parahoric subgroup  $\mathcal{P}$  of  $G_F$ . On the other hand, for a local field  $E$

of characteristic 0 with the same residue field  $k$ , we can naturally associate a parahoric subgroup  $\mathcal{P}'$  of  $G_E$  to  $\mathcal{P}$ , and consider the Schubert variety over  $k$  corresponding to  $\mu$  in the Witt vector affine Grassmannian of  $\mathcal{P}'$ . In this talk, we show that if the absolute ramification index of  $E$  is sufficiently large, then the Schubert varieties corresponding to  $\mu$  in these two settings are isomorphic to each other (after taking perfection). This result aligns with the Deligne-Kazhdan philosophy of close fields. For hyperspecial subgroups, this result was proved by Bando. Our approach is different from that of Bando: we establish a certain representability result for a version of the affine Grassmannian defined using the moduli space of non-archimedean local fields introduced by Li-Huerta, from which we deduce our result. If time permits, I will also discuss potential applications to orbital integrals for  $G_F$ . This talk is based on joint work with Sebastian Bartling.

## Moduli spaces of analytic $p$ -divisible groups

Lucas Gerth

Institut de Mathématiques de Jussieu - Paris Rive Gauche, France

We prove a classification of families of analytic  $p$ -divisible groups on adic spaces  $S$  over  $\mathbb{Q}_p$  in terms of Hodge-Tate triples on  $S$ , generalizing a theorem of Fargues. From this, for  $S$  a perfectoid space, we construct an analytic Dieudonné theory with values in mixed characteristic Shtukas over the Fargues-Fontaine disc. As applications, we realize the local Shimura varieties of EL and PEL type of Rapoport-Zink as moduli spaces of analytic  $p$ -divisible groups with framed universal cover, and we reinterpret the Hodge-Tate period map of Scholze in terms of  $p$ -topological torsion subgroups of abelian varieties.

## Raynaud-style semistability for degenerating families

David Urbanik

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From the perspective of semistability, the theory of abelian varieties is special, in that the weight filtration appearing in étale cohomology has a geometric rigid-analytic origin (a theorem of Raynaud). We describe work in progress aimed at generalizing this perspective to arbitrary rigid-analytic degenerations of algebraic varieties to a normal crossing divisor.

## On the geometric side of the Guo-Jacquet trace formula

Huajie Li (李华杰)

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A well-known theorem of Waldspurger relates the central value of automorphic L-function for  $GL(2)$  to the period integral on a nonsplit torus. Guo-Jacquet proposed a relative trace formula approach to generalise this theorem to higher dimensions. In a joint work in progress with Pierre-Henri Chaudouard, we establish such a trace formula in the coarse form. We shall explain the

background and present our results with emphasis on the geometric side, where we obtain relative weighted orbital integrals for most terms.

## **$p$ -adic Periods of CM Abelian Varieties**

**Roy Zhao( 赵世豪 )**

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While the Colmez Conjecture is often stated as an equality between the Faltings heights of CM abelian varieties and special values of logarithmic derivatives of  $L$ -functions, it was originally formulated as a product formula for  $p$ -adic periods of CM abelian varieties. In fact, proving the equality between Faltings heights and special values of  $L$ -functions will only give "half" of Colmez's original conjecture. We will explain how this product of  $p$ -adic periods can be understood in terms of Arakelov geometry. This geometric reformulation gives an avenue to tackle the full Colmez Conjecture. As a consequence, we show that the Colmez Conjecture can be reduced to a height formula for surfaces, and recover some previous known results concerning heights of nearby CM-types.

## **A multiplicity one result in $\text{mod } p$ Jacquet-Langlands correspondence for $\text{GL}_2(\mathbb{Q}_p)$**

**Yongquan Hu( 胡永泉 )**

Morningside center of Mathematics, AMSS, CAS, China

The classical Jacquet-Langlands (J-L) correspondence relates complex smooth representations of  $\text{GL}(n)$  and that of its inner forms. In the  $\text{mod } p$  setting, Scholze constructed a cohomological covariant  $\delta$ -functor from the category of admissible smooth  $\text{mod } p$  representations of  $\text{GL}_n(L)$  to admissible smooth  $\text{mod } p$  representations of  $D^\times$  which carry a continuous Galois action. In this talk I will report some results on Scholze's functors in the (simplest) case of  $\text{GL}(2, \mathbb{Q}_p)$ ; in particular I will focus on a multiplicity one property related to Scholze's functors and its applications. This is joint work with Haoran Wang.

## **Geometric presentations and torsors over affine lines**

**Ning Guo( 郭宁 )**

Harbin Institute of Technology, China

Geometric presentations are an effective method for simplifying cohomological problems by reducing their relative dimension. This talk will trace the history of this technique, from Artin's "bon voisinage" to its modern applications. We will particularly focus on its relevance to the Grothendieck-Serre conjecture for reductive torsors over regular local rings. In this context, the talk will also review recent advances in the analysis of torsors over affine and projective lines.

## Prismatic-étale comparison theorem and $p$ -adic nearby cycles for semistable local systems

Yichao Tian(田一超)

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Let  $K$  be a finite extension of  $\mathbb{Q}_p$ , and  $X$  be a semistable  $p$ -adic formal scheme over  $\mathcal{O}_K$ . Semistable étale local systems on  $X_K$  can be viewed as natural generalizations of classical semistable Galois representations over  $K$ . Recently, Du-Liu-Moon-Shimizu proved that the category of semistable étale  $\mathbb{Z}_p$ -local systems on  $X_K$  is equivalent to analytic prismatic  $F$ -crystals on the absolute log-prismatic site of  $X$ . In this talk, I will explain a comparison theorem between the geometric étale cohomology of a semistable  $\mathbb{Z}_p$ -local system on  $X_K$  and the cohomology of its attached log-prismatic  $F$ -crystal.

## Prismatic cohomology and Hopf algebroids

Jingbang Guo(郭靖邦)

Shanghai Center for Mathematical Sciences, Fudan University, China

The prismatic cohomology, introduced first by Bhatt and Scholze, is a cohomology theory for  $p$ -adic formal schemes which can be specialized to many other cohomology theories. In this talk, following the idea of prismatization, we will briefly explain the method of studying prismatic cohomology by using suitable Hopf algebroids. Then we apply this method to study prismatic cohomology of polynomial rings: by considering certain  $q$ -de Rham prisms and corresponding Hopf algebroids, we obtain explicit complexes (with  $q$ -de Rham operators) computing prismatic cohomology.

This talk is based on joint work with Ruochuan Liu and Guozhen Wang.

## On the homological local model theorem

Zhongyipan Lin(林中一攀)

Tongji University, China

In this talk, I will report on recent advancements of the geometric Breuil-Mezard conjecture. First, I will explain how to use the homological local model theorem to promote the topological Breuil-Mezard conjecture to the geometric Breuil-Mezard conjecture. Then I will talk about the correct form of the topological Breuil-Mezard conjecture. Finally, I will say a few words about the proof of the homological local model theorem for small Hodge types and generic Weil-Deligne types. This is partly based on joint work in progress with Tony Feng and Bao Le Hung.

## Serre weight conjectures and modularity lifting for $\mathrm{GSp}_4$

Heejong Lee

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Given a Galois representation attached to a regular algebraic cuspidal automorphic representation, the Hodge-Tate weight of the Galois representation is matched with the weight of the automorphic representation. Serre weight conjectures are mod  $p$  analogue of such a correspondence, relating ramification at  $p$  of a mod  $p$  Galois representation and Serre weights of mod  $p$  algebraic automorphic forms. In this talk, I will discuss the proof of the conjecture for the group  $\mathrm{GSp}_4$  using the geometry of " $\ell = p$ " local Galois representations and representation theory of finite groups of Lie type (e.g.  $\mathrm{GSp}_4(\mathbb{F}_p)$ ). If time permits, I will explain how one can deduce modularity lifting theorem by combining our result with the recent progress on the Breuil-Mézard conjecture by Feng-Le Hung-Lin. This is based on a joint work with Daniel Le and Bao V. Le Hung.

## A Prismatic Herr Complex for Bloch-Kato Selmer Groups

Luming Zhao( 赵路明 )

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In 1998, Laurent Herr constructed a three-term complex using étale  $(\phi, \Gamma)$ -modules to compute the Galois cohomology of corresponding Galois representations. This approach was later extended to various settings, and recently to Kummer setting via  $(\phi, \tau)$ -modules. Inspired by the work of Du-Liu, we will introduce a prismatic Herr complex, formulated using a prismatic version of  $(\phi, \tau)$ -modules attached to a (log-)crystalline representation. For a crystalline representation, this complex computes the Bloch-Kato Selmer group and relates to the cohomology of the corresponding  $F$ -gauge. The talk is based on joint work with Heng Du.

## On $\mathbb{C}_p$ representations of the fundamental group in the $p$ -adic Simpson correspondence over curves

Daxin Xu( 许大昕 )

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In the  $p$ -adic Simpson correspondence over a smooth proper curve, an important question is to understand the essential image of the continuous  $\mathbb{C}_p$  representations of the geometric fundamental group under this correspondence. The essential image is expected to consist of semistable Higgs bundles of degree zero. We first review the work of Deninger--Werner and myself on this question. We then discuss some recent progress, including the case of rank two Higgs bundles with a non-zero nilpotent Higgs field.