Nonlinear free-surface flows close to the limiting configuration with a 120 Stokes angle

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Many two-dimensional steady nonlinear potential free-surface flows have limiting configurations with a 120 Stokes angle. These include travelling waves, flows past submerged obstacles and rising jets. In this talk we study these flows just before they reach the limiting configuration. The 120 degree angle is then replaced by a free-surface with an arbitrary large curvature. We show that the corresponding flow is then described by a self similar solution first derived by Longuet-Higgins (1978). The numerical computations use a simple method based on an extension of the works of Mitchell and Havelock. A detailed study of the limiting configurations is presented. Extensions to interfacial waves are also described. This work is joined with Jason Chan and Xin Guan.

Nonlinear, but under control: a hierarchical modelling approach to manipulating waves in fluids

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Falling liquid films represent a canonical physical scenario underpinned by complex processes that can be modelled using infinite-dimensional dynamical systems. The interfacial behaviour is characterised by the Reynolds number, a non-dimensional parameter; above a critical value the uniform film becomes unstable, leading to the emergence of nonlinear travelling waves. By injecting and removing fluid from the base at discrete locations, we aim to stabilise otherwise unstable interfaces towards target states (e.g. flat interfaces), with the additional limitation that observations of the state are restricted to finitely many measurements of the film height. Suc-cessful feedback control has recently been achieved in the case of full observations using linear-quadratic regulator controls coupled to asymptotic approximations of the Navier-Stokes equations (Holroyd, Cimpeanu & Gomes, SIAM J. Appl. Math., 2024), but restricted observations severely curtailed their performance. We proceed to couple the well-understood full-information feedback control strategy to a non-linear estimator. The dynamics of the estimator are designed to approximate those of the film, and we apply a forcing term chosen to ensure that measurements of the estimator match the available measurements of the film. Using this method, we restore the performance of the controls to a level approaching their full-information counterparts, even at moderately large Reynolds numbers. We also briefly investi-gate the effects of noise and the relative positioning of actuators and observers on the resulting dynamics (Holroyd, Cimpeanu & Gomes, Royal Soc. Proc. A, 2025). This is a joint work with Oscar Holroyd and Susana N. Gomes (U. Warwick).

Keywords: stratified flows, non-local asymptotics, thin film dynamics, instability, direct numerical simulations, volume-of-fluid method

Water-wave fields reconstruction via space transformation method and metamaterials

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Manipulating water waves and reconstruct their fields have important application prospects in ocean engineering. The space transformation approach, based on the form-invariance of the Helmholtz equation, provides a framework for the mathemat-ical design of shallow-water wave fields, with physical realization achievable through metamaterials. In this study, to manipulate wave characteristics such as path, wave-length, and height, several wave fields were constructed using the space transforma-tion method. To this end, anisotropic water depth and spatial gravitational ac-celeration were introduced into the Helmholtz equation. Next, metamaterials were employed to physically realize the designed anisotropic media. The efficacy of these metamaterials in reconstructing the target wave fields was then confirmed by solving the Navier–Stokes equations. This validation showed that the method successfully reconstructs shallow-water wave fields mathematically, leading to controlled changes in energy distribution and structural hydrodynamic properties.

Keywords: Water waves, Helmholtz equation, Space transformation method, Meta-material

Rogue waves in complex sea states

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Extreme ocean wave events, known as rogue waves (RWs), pose significant risks to human activities and infrastructure near the ocean surface. These events can arise from either wave turbulence or the emergence of coherent structures. While the theory of wave turbulence, which assumes linear superposition and random Fourier phases, has been extensively studied, the mechanisms behind coherent structures, which are primarily driven by three- and four-wave interactions depending on water depth, remain an open question, particularly in directional or crossing seas. In this presentation, we recall our previous works and report observations of coherent RW structures in three distinct and related scenarios: deep-water standing waves with a specific crossing angle of 180 degrees, deep-water crossing seas with arbitrary angles, and realistic crossing seas in coastal regions. Our findings underscore the critical role of nonlinear coherent structures in RW formation, which could enhance the accuracy of RW predictions and inspire related interdisciplinary research fields.

A novel Kadomtsev–Petviashvili type model for nonlinear internal waves with horizontally two-dimensional shear currents and Earth's rotation

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Inspired by the need to theoretically understand the naturally occurring interactions between internal waves and mesoscale phenomena in the ocean, we derive a novel model equation from the primitive rotational Euler equations using the multi-scale asymptotic expansion method. By applying the classic balance $\epsilon = \mu^2$ between nonlinearity (measured by ϵ) and dispersion (measured by μ), along with the assumption that variations in the transverse direction are of order μ, which is smaller than those in the propagation direction, we arrive at terms from the classic Kadomtsev-Petviashvili equation. However, when incorporating background shear currents in two horizontal dimensions and accounting for Earth's rotation, we introduce three additional terms that, to the best of the authors' knowledge, have not been addressed in the previous literature. Theoretical analyses and numerical results indicate that these three terms contribute to a tendency for propagation in the transverse direction and an overall variation in wave amplitudes. The specific effects of these terms can be estimated qualitatively based on the signs of the coefficients for each term and the characteristics of the initial waves. Finally, the potential shortcomings of this proposed equation are illuminated.

Obliquely interacting solitary waves and wave wakes in free-surface flows

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Mach reflection is a crucial phenomenon in aerodynamics, and similar occurrences are observed during the propagation and evolution of solitons in the ocean. This report is centered on the weakly nonlinear isotropic bidirectional Benney-Luke (BL) equation, which is employed to characterize surface waves and internal waves in the ocean. By employing the classic fourth-order Runge-Kutta method, the pseudo-spectral scheme with the integrating factor method, and the windowing scheme, the numerical simulation of the evolution process of V-shaped and reverse V-shaped solitons at various angles is conducted. The investigation reveals that when the angle of the V-shaped soliton is significant, Mach reflection occurs: a small-amplitude soliton, perpendicular to the propagation direction, is generated at the discontinuity point, which is the so-called Mach stem. As the angle of the V-shaped soliton gradually decreases to a critical angle, the amplitude of the Mach stem diminishes to zero. When the angle of the V-shaped soliton is less than the critical angle, regular reflection takes place, with an arc-shaped wave connecting the two branches of the V-shaped soliton. As the wave evolves, the radius of the arc-shaped wave progressively increases while the amplitude gradually decreases until it vanishes. In the case of the reverse V-shaped soliton, when the angle of the soliton is significant, Mach reflection occurs: a large-amplitude Mach stem is generated at the discontinuity point, along with two symmetrical small-amplitude solitons. As the angle of the solitons decreases to the critical angle, the amplitude of the Mach stem further increases, yet the rate of growth of the Mach stem length gradually diminishes to zero. When the angle of the reverse V-shaped soliton is less than the critical angle, regular reflection takes place, ultimately forming an X-shaped soliton. Through the Whitham modulation theory, modulation equations for the amplitude and slope of solitons are constructed, providing analytical expressions for the critical angle of Mach reflection and the amplitude of the Mach stem. A comparison with the Kadomtsev–Petviashvili

(KP) equation reveals that as the soliton velocity approaches 1, the conclusions drawn from the BL equation align more closely with those of the KP equation.

Diffraction and interaction of interfacial solitons in a two-layer fluid of great depth

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We present a novel isotropic bi-directional model for studying weakly dispersive and weakly nonlinear internal waves in a three-dimensional system consisting of two superimposed, incompressible, and inviscid fluids. The newly developed equa-tion is the Benjamin–Benney–Luke (BBL) equation, a generalization of the famous two-dimensional Benjamin–Ono (2DBO) equation and the Benney–Luke equation, derived using the nonlocal Ablowitz–Fokas–Musslimani formulation of water waves. The evolution results of the BBL and 2DBO equations, performed by implement-ing the classic fourth-order Runge–Kutta method, the pseudo-spectral scheme with the integrating factor method, and the windowing scheme, show that the anisotropic 2DBO equation agrees well with the isotropic BBL model for problems being investi-gated, namely the focus is the central part of the soliton evolution/interaction zone. By applying the Whitham modulation theory, modulation equations for the 2DBO equation are obtained for analyzing the soliton dynamics in different initial-value problems. In addition, corresponding numerical results are obtained and shown to agree well with the theoretical predictions. Both theoretical and numerical re-sults reveal the formation conditions of the Mach expansion, as well as the specific relationship between the amplitude of the Mach stem and the initial data.

Two-layer interfacial waves: linear stability and ocean surface wave breaking

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Interfacial waves in air–sea interaction profoundly influence mixing in the oceanic boundary layer, modulating exchanges of momentum, heat, and freshwater that are essential to Earth's weather and climate. This study examines two-layer interfacial waves from two perspectives: (1) the linear stability of two-layer inviscid and viscous flows, and (2) the physical energetics of the surface-breaking process.

We first analyze the stability of an inviscid two-layer rotating flow with a flat interface and find instability. We then examine the linear stability of a two-layer vis-cous stagnation-point flow with a similarity-solution base state under the long-wave approximation. Properties of the system and several limiting flow configurations are discussed. As an application to ocean dynamics, we employ multiphase direct numerical simulations of deep-water surface-breaking waves by solving the incom-pressible Navier–Stokes equations with surface tension. By varying wave steepness and the Bond number, we analyze the energy budget to reveal how wave energy is redistributed into turbulent dissipation, bubble generation, and droplet dynamics.

Modelling perturbed long plane, ring and hybrid surface waves

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The two-dimensional (2D) evolution of perturbed long weakly-nonlinear surface plane, ring, and hybrid waves, consisting, to leading order, of a part of a ring and two tangent plane waves, are modelled numerically within the scope of the 2D Boussinesq-Peregrine system. Numerical runs are initiated and interpreted using the reduced 2D cylindrical Korteweg-de Vries (cKdV)-type and Kadomtsev-Petviashvili II (KPII) equations. The cKdV-type equation leads to two different models, the $\mathrm{KdV}\theta$, where θ stands for a polar angle, and cKdV equations, depending on whether we use the general or singular (i.e. the envelope of the general) solution of the associated nonlinear first-order differential equation. The $\mathrm{KdV}\,\theta$ equation is also derived directly from the 2D Boussinesq-Peregrine system and used to analytically describe the intermediate 2D asymptotics of line solitons subject to sufficiently long transverse perturbations of finite strength, while the cKdV equation is used to initiate outward- and inward-propagating ring waves with localised and periodic perturbations. Both of these equations, together with the KPII equation, are used to model the evolution of hybrid waves, where we show, in particular, that large localised waves (lumps) can appear as transient (emerging and then disappearing) states in the evolution of inward-propagating waves, contributing to the possible mechanisms for the generation of rogue waves. Detailed comparisons are made between the key features of the non-stationary two-dimensional modelling and relevant predictions of the reduced equations. This is joint work with Karima Khusnutdinova and Dmitri Tseluiko.

Nonlinear long waves over bottom topography

Wooyoung Choi
New Jersey Institute of Technology, United States

Generation and propagation of nonlinear long waves over bottom topography are studied using high-order strongly nonlinear long wave models. Both steady and unsteady wave solutions of the models are computed and compared with those of the Euler equations to validate the models. Special attention is paid to the solution dependence on the order of nonlinearity and the Froude number.

Conformal mapping technique for modeling nonlinear free-surface or interface waves

Yuriy Semenov
Harbin Engineering University, China

A conformal mapping technique for solving two-dimensional nonlinear potentialflow problems involving free or interfacial boundaries is presented. These include free-surface waves, capillary waves, and flexural-gravity waves generated by the interaction of a uniform current with a submerged body, complex bottom topography in a channel, or another fluid. When solving such

problems, it is necessary at some stage to determine a function, such as the complex potential, the mapping function, or the complex velocity, from its values prescribed by the boundary conditions along the entire boundary of the flow domain.

In this talk we briefly review the historical progress in solving nonlinear boundaryvalue problems based on the development of complex-variable function theory. This begins with Kirchhoff's conformal mapping technique based on a conformal transformation between two geometric regions; continues with the Joukowskii-Michell method, which introduced the concept of an auxiliary parameter plane (the ζ-plane) and employed the Schwarz-Christoffel integral; and includes Chaplygin's method of special points, which allows one to obtain the desired complex function without explicitly using a conformal mapping. This method consists in determining all zeros and poles of a complex function by analysing its behaviour in the physical flow region. Then, applying the Riemann-Schwarz symmetry principle and Liouville's theorem, the desired function is obtained.

In this talk we show how to apply Chaplygin's special-points technique to more complicated problems that account for variation of the slope along the body and variation of the velocity magnitude on the free surface or interface due to gravity, surface tension, interaction with an elastic plate, or another liquid. In other words, we show how to derive integral formulae that determine a complex function from its values on the boundary of a parameter region.

Let the first quadrant be chosen as the definition domain of an analytic function $F(\zeta)$, where $\zeta = \xi + i\eta$. Suppose that the argument $\beta(\xi) = \arg(F(\xi))$ is known on the real axis of the first quadrant, and the modulus $\rho(\eta) = |F(i\eta)|$ is known on the imaginary axis of the first quadrant. Then the following formula determines the complex function $F(\zeta)$:

$$F(\zeta) = \rho_\infty \exp\left[\frac{1}{\pi}\int_0^\infty \frac{d\beta}{d\xi}\ln\left(\frac{\zeta+\xi}{\zeta-\xi}\right)d\xi - \frac{i}{\pi}\int_0^\infty \frac{d(\ln\rho)}{d\eta}\ln\left(\frac{\zeta-i\eta}{\zeta+i\eta}\right)d\eta + i\beta_\infty\right], \tag{1}$$

where $\beta_\infty = \lim_{\xi\to\infty}\beta(\xi)$, $\rho_\infty = \lim_{\eta\to\infty}\rho(\eta)$. Equation (1) may be verified: for $\zeta = \xi$, its argument equals $\beta(\xi)$, and for $\zeta = i\eta$, its modulus equals the prescribed function $\rho(\eta)$.

The integral equations (1) have been widely used to solve nonlinear free-boundary problems. In this talk, special attention is given to the nonlinear analysis of flexuralgravity waves arising from the interaction of a uniform current with a submerged body [1], an obstruction on the channel bottom [2], or a channel bottom with complex geometry [3]. We present case studies for a widened rectangular obstruction on the channel bottom, whose width exceeds the wavelength of the interface, and for arrays of triangular ripples forming the undulating bottom shape. The influence of the bottom shape on the interface is considered for three flow regimes: the subcritical regime, $F < F_{cr}$, for which the depth-based Froude number is less than the critical Froude number, and the interface perturbation decays upstream and downstream of the obstruction; the ice-supercritical and channel-subcritical regime, $F_{cr} < F < 1$, for which two waves of different wavelengths extend upstream and downstream to infinity; and the channel-supercritical regime, $F > 1$, for which the hydroelastic wave extends downstream to infinity. The results revealed a trapped interface wave above the rectangular obstruction and the ripple patch. The resonance behaviour of the interface over the undulating bottom occurs when the period of ripples approaches the wavelength of the ice/liquid interface.

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Exact solutions and short-wavelength instability for geophysical waves at arbitrary latitude

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We survey some exact solutions in the Lagrangian framework, representing waves at arbitrary latitude that propagate eastward or westward above a flow which accom-modates a constant underlying background current, waves that can be both in the direction of the current and in the opposite direction. These waves are linearly unsta-ble to short-wavelength perturbations, if their steepness exceeds a specific threshold. This threshold depends on the latitude and the strength of the underlying current.

Nonlinear tension effects on an infinite elastic plate loaded with a linear shear current

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This work examines the dynamics of a fluid-loaded thin elastic plate, with a particular focus on the effects of nonlinear tension. While previous research concentrated on fixed-wavelength periodic waves with a uniform background current, our research broadens this framework to consider general periodic waves propagating along an infinite plate influenced by a linear shear current. Moreover, we present new rigorous formulations to address the structural nonlinearities of the plate. Given that the tension coefficient (denoted as $\mathcal{N}$) can be large in practical applications, we explore the system's modulational behavior across three distinct tension coefficient regimes. In the case of small $\mathcal{N}$, an additional integral term emerges in the nonlinear Schrödinger equation; however, it does not influence the modulational instability. For intermediate values of $\mathcal{N}$, the energy exchange during three-wave resonance is slowed down. In scenarios with large $\mathcal{N}$, we identify a nonlinear dispersion relation along with an amplitude-dependent speed for wavepacket propagation. To analyze the full Euler equations, we utilize the conformal mapping method in conjunction with an iterative scheme for numerical computations. The results of direct numerical simulations align well with our theoretical analyses.

Generation of deterministic and directional rogue waves in a basin

Amin Chabchoub
Okinawa Institute of Science and Technology, Japan
The University of Melbourne, Australia

Rogue waves pose a significant threat to marine vessels and offshore infrastructure, making a clear understanding of their dynamics and kinematics essential. While numerical and experimental modeling plays a crucial role in this effort, an ongoing debate within the research community concerns the relative importance of nonlinear-ity and unstable wave groups in the formation of realistic, directional rogue waves. This talk addresses this issue by highlighting the pivotal role of weakly nonlinear wave theory in the generation and control of deterministic and directional rogue waves, both in the open ocean and in controlled water-wave basin experiments.

Nonlinear deep water waves on shoaling area based on NLS

Zhen Wang
Beihang University, China

Waves on shoaling area shows profile transformation, including the nonlinear steep crest and dispersion, even breaking, the energy and amplitude are also changes along varying topography. Nonlinear schrodinger equation can describe the deep water wave evolution and propagation on shoaling area in terms of intermediate depth water, even the critical depth $kh = 1.363$. the statistics values of random waves are analyzed for bimodal spectrum. The probability of rogue wave are also considered through numerical simulation.

Nearshore bathymetry inversion and wave fields reconstruction based on physics-informed neural networks

Zhengzhi Deng
Zhejiang University, China

High-precision marine bathymetry plays a crucial role in coastal engineering, ma-rine hazard warning, and ecological restoration. However, conventional measure-ment methods, constrained by their reliance on dense data collection, high oper-ational costs, and limited resolution, often fail to meet the increasing demand for high-precision and continuous observation. In this study, we encode energy balance equations, nonlinear dispersion relations, and shallow water equations into a neural network to develop a wave-depth coupled multi-physics inversion model based on Physics-Informed Neural Networks (PINNs). The model is designed to evaluate the performance of PINNs in bathymetry inversion and wave field reconstruction under varying incident wave frequencies. Furthermore, a comparative analysis is carried out to assess the error distributions in wave height reconstruction and depth inver-sion when linear versus nonlinear dispersion relations are embedded in the PINNs framework, thereby elucidating the influence of dispersion effects on model accuracy. The results demonstrate that PINNs, by deeply integrating neural networks with physical governing equations, exhibit strong adaptability and robustness in complex nearshore

environments. This technical framework can be extended to other marine parameter inversion tasks, offering an innovative approach for coastal engineering monitoring, marine hazard early warning, and ecological restoration projects.

Faraday instability based on a gap-resolved method

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Current theoretical analyses of Faraday instability in a Hele-Shaw cell typically em-ploy gap-averaged governing equations and rely on the Hamraoui model derived from molecular kinetic theory. This approach oversimplifies gap wise information such as the contact line velocity and capillary hysteresis, leading to contradictions with experimentally observed unsteady meniscus dynamics and pinned contact line behavior. In this paper, by directly solving the transverse gap flow and adopting a modified contact angle hysteresis model, we propose a gap resolved method that successfully overcomes the limitations of previous technical routes. Ultimately, a modified damping coefficient related to the static contact angle and hysteresis range is obtained. By combining this damping with the gap averaged damping from Stokes boundary layer theory and the viscous dissipation within the fluid domain, we derive a new amplitude equation for Faraday waves. Using Lyapunov's first method, an an-alytical expression for the critical stability boundary is obtained. Moreover, unlike previous studies that directly employ empirical parameters, the contact angle pa-rameters required for the theoretical predictions in this work are entirely determined through experimental measurements. Experimental comparisons over a wide range of driving frequencies validate the improvement of the proposed model in addressing Faraday instability problems at gap interfaces.

Evolution of capillary-gravity waves under the action of wind and dissipation

Wenhao Cheng
Huazhong University of Science and Technology, China

This study investigates the nonlinear evolution of capillary-gravity waves in deep water under wind forcing and dissipation. Previous research has identified an ini-tial stage of amplitude growth followed by frequency downshift during small-scale wind wave evolution. Here, we reveal a new intermediate stage between these two established phases. This stage is excited by coupled three-wave and four-wave near-resonant interactions, leading to spectral broadening. During this period, the total wave energy may temporarily decrease due to wave breaking. As the dominant com-ponent shifts to lower frequencies, the evolution enters the third stage, characterized by a continuous increase in total wave energy and a sustained downshifting of the dominant frequency.

Shoaling of internal solitary waves over rough sloping topography

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Internal solitary waves (ISWs) are large-amplitude, nonlinear disturbances that propagate along density interfaces in stably-stratified fluids such as oceans and lakes. Occurring at depth along the pycnocline, they can travel hundreds of kilometers without losing shape. Generated by large-scale forcing such as tides and wind, ISWs transfer energy from basin-scale motions to smaller scales where it is dissipated, driv-ing diapycnal mixing that influences global circulation and biogeochemical cycles. They also impact offshore structures, navigation, and acoustic propagation, while interactions with the seabed resuspend sediments and alter benthic habitats.

As ISWs shoal over continental slopes, they steepen, break, and dissipate en-ergy, enhancing mixing and sediment transport. Mode-1 ISWs of depression are most common and often fission into elevation waves during shoaling. Breaking types – plunging, collapsing, surging, and fissioning – have been well studied over smooth, idealised slopes, but natural slopes are rarely uniform. Roughness elements such as ripples, rocks, and vegetation can significantly modify breaking behaviour, turbulence generation, and mixing efficiency, yet their role remains underexplored.

This study uses the Spectral Parallel Incompressible Navier–Stokes Solver (SPINS) to investigate ISW shoaling over rough sloping topography across all breaking regimes. Simulations span slope steepnesses typical of continental margins and lakes, with fixed stratification and Reynolds number, and are validated against previous exper-imental and numerical work on smooth slopes and rough horizontal beds. Results show that bottom corrugations alter wave shape and induce Kelvin–Helmholtz bil-lows in plunging and collapsing cases; impede upslope propagation and generate reflected mode-2 waves in surging scenarios; and locally modify layer depths during fission, potentially arresting and restarting the process. Despite these mechanical effects, analysis of density and enstrophy fields via a user-defined scatter plot (USP) method indicates that post-wave mixing distributions remain largely unaffected, sug-gesting roughness exerts a transient influence confined to the active shoaling region – though in fissioning cases, this influence may extend across much of the slope.

The long-wave vorticity dynamics of coastal fronts

Ted Johnson
University College London, United Kingdom

This talk discusses the propagation of free, long waves on a potential vorticity front in the presence of a vertical coast, using a 1.5 layer, quasi-geostrophic model with piecewise-constant potential vorticity. The coastal boundary induces flow through image vorticity and a Kelvin wave, either of which can reinforce or oppose the Rossby wave dynamics at the front. The behaviour of the front depends strongly on the relative strengths of these three mechanisms, which are explicit in the model. The richest behaviour, which includes kink solitons (under-compressive shocks) and compound-wave structures, occurs in the regime where vortical effects are dominant. The evolution of the front is described by a fully nonlinear finite-amplitude equation including first-order dispersive effects, which is related to the modified Korteweg–de Vries equation. The different behaviours are classified using the canonical example of the Riemann problem, which can be described using El's technique of 'dispersive shock-fitting'. Contour-dynamic simulations show that the dispersive long-wave theory captures the behaviour of the full quasi-geostrophic system to a high degree of accuracy.

Open channel flow over topography

Mark Blyth
University of East Anglia, United Kingdom

We discuss two-dimensional free-surface flow in an open channel with a compact topographic feature on the bottom. In the first part of the talk, we will discuss the inverse problem of inferring the bottom geometry from the surface profile. Under a commonly made rule-of-thumb approximation the inverse problem has an exact solution whose existence we discuss with reference to certain singularities in the complex plane. In the second part of the talk, we revert to the forwards problem and discuss the stability of a hydraulic fall flow over a local depression in the bottom. In particular we present numerical evidence of an apparently new type of unsteady free-surface flow that is time-periodic and which is characterised by a localised pulsation above the depression with ripples emanating from either side of the hydraulic fall and propagating into the far field.

Viscous-dispersive shock profiles for isentropic compressible fluids of Korteweg type

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Universidad Nacional Autónoma de México, Mexico|

In this talk I discuss the existence and stability properties of viscous-dispersive shock profiles for isentropic compressible fluids of Korteweg type. The system describes the dynamics of a compressible isentropic fluid exhibiting viscosity and internal capillarity in one space dimension and in Lagrangian coordinates. It is assumed that the viscosity and the capillarity coefficients are very general nonlinear smooth, positive functions of the specific volume. It is shown that the system admits traveling wave solutions connecting two constant states and traveling with a certain speed that satisfy the classical Rankine-Hugoniot and Lax entropy conditions. These traveling wave solutions are unique up to translations and have arbitrary amplitude. The spectral stability of such viscous-dispersive profiles is also considered. It is shown that the essential spectrum of the linearized operator around the profile (posed on an appropriate energy space) is stable, independently of the shock strength. With the aid of energy estimates, it is also proved that the point spectrum is also stable, provided that the shock amplitude is sufficiently small and a structural condition on the underlying inviscid shock is fulfilled. This is joint work with Corrado Lattanzio (University of L'Aquila, Italy) and Raffaele Folino (Universidad Nacional Autónoma de México).

Impact of mean water level on particle drift in shallow and intermediate depth

Philippe Guyenne
University of Delaware, United States

Particle motions under nonlinear gravity waves at the free surface of a two-dimensional incompressible and inviscid fluid are considered. The Euler equations are solved numerically using a high-order spectral method based on a Hamiltonian formulation of the water wave problem. Extending this approach, a numerical procedure is devised to estimate the fluid velocity at any point in the fluid domain given surface data. The reconstructed velocity field is integrated to obtain particle trajectories for which an analysis is provided, focusing on two questions. The first question is the influence of a wave setup or setdown as is typical in coastal conditions. It is shown that such local changes in the mean water level can lead to qualitatively different pictures of the internal flow dynamics. These changes are also associated with rather strong background currents which dominate the particle transport and, in particular, can be an order of magnitude larger than the well-known Stokes drift. The second question is whether these particle dynamics can be described with a simplified wave model.

The Korteweg-de Vries equation is found to provide a good approximation for small- to moderate-amplitude waves on shallow and intermediate water depth. Despite discrepancies in severe cases, it is able to reproduce characteristic features of particle paths for a wave setup or setdown. This is joint work with Henrik Kalisch (U. Bergen).

Rediscovering shallow water equations from experimental data

Mats Ehrnström
Norwegian University of Science and Technology, Norway

New data-driven methods have advanced the discovery of governing equations from observations, enabling parsimonious models for complex systems. Here, we 'rediscover' a shallow-water equation closely related to Korteweg-de Vries (KdV) using only video recordings of solitons in a simple flume. Two fundamentally different approaches - weak-form sparse identification of nonlinear dynamics (WSINDy) and a novel Fourier-multiplier method - recover the same PDE, demonstrating that the equation is inherent in the data and robust to the choice of method. Both identify the same terms with comparable magnitudes and errors. To validate the models, we solve the discovered equations forward in time and compare them with additional experimental cases that were not used in the discovery. Based on the results, we discuss absolute and cumulative errors, as well as the strengths and limitations of the two discovery approaches. Together, these results demonstrate the potential of equation discovery from everyday experiments ('GoPro physics') and highlight shallow-water waves as an ideal test bed for developing and benchmarking new methods.

Air-blown waves on viscous liquid films: steady solutions, dynamics and the potential control strategy

Yanghan Meng
Imperial College London, United Kingdom

Thin viscous liquid films flowing down an inclined wall under gravity in the presence of an upward-flowing high-speed air stream are considered. The air stream induces nonlinear interfacial waves, and asymptotic analysis is used to derive a non-local evolution equation forced by analytically obtained air-pressure effects and incorporating a constant tangential stress. Benney equations in the capillary (strong surface tension) and inertio-capillary regimes are derived and studied. The air stream produces Turing-type short-wave instabilities in sub-critical Reynolds number regimes that would be stable in the absence of the outer flow. Extensive numerical experiments are carried out to elucidate bifurcation of steady solutions and the rich dynamics in the above-mentioned short-wave regime. Numerical computations show that there exist two distinct primary bifurcation branches of steady solutions starting from infinitesimal waves. New families of solutions manifest themselves either as secondary bifurcation occurring on primary branches or as isolated solution branches. Time-dependent nonlinear computations are also performed to track the large-time behaviour of attractors. A fairly complete picture of different solution types is categorised in parameter space. Finally, Benney equations are extended by incorporating the influence of periodic point vortices. Numerical results demonstrate that appropriate vortex forcing through its strength and periodicity can synchronize distinct attractors into stable periodic waves, suggesting a feasible mechanism for interfacialwave control. This is a joint work with Demetrios Papageorgiou (Imperial College) and Jean-Marc Vanden-Broeck (University College London).

A numerical study on 3D gravity-capillary standing waves

Xin Guan
Imperial College London, United Kingdom

In this talk, we present a numerical study of 3D gravity-capillary standing waves by using cubic and quintic truncated Hamiltonian formulations and Craig-Sulem expansion of the Dirichlet-Neumann operator. The resulting models are treated as triply periodic boundary-value problems and solved via Newton's method without evaluating initial-value calculations. This approach avoids numerical stiffness associated with surface tension and numerical instabilities arising in time integration. We reduce the number of unknowns significantly by exploiting the spatial-temporal symmetries for three types of symmetric standing waves. Comparisons with existing asymptotic and numerical results illustrate excellent agreement between the models and the full potential-flow formulation. We investigate typical bifurcations and standing waves that feature square, hexagonal, and nearly two-dimensional patterns under three-wave resonance. These solutions are generalisations of the classical Wilton ripples. Temporal simulations of the computed three-dimensional standing waves reveal perfect periodicity in time and an instability mechanism based on the reported oblique instability for two-dimensional standing waves.

Multi-solitary-wave solutions of a model equation for water waves

Shuming Sun
Virginia Polytechnic Institute and State University, United States

The talk concerns the existence of multi-hump solutions, also called multi-solitarywave solutions, for a model equation of water waves - a singularly perturbed Kortewegde Vries (KdV) equation. The equation has been used to study the surface waves on water of finite depth with small surface tension on the free surface. It was known that the equation has homoclinic solutions connected to small oscillatory tails at infinity, called generalized solitary-wave solutions (or generalized one-hump solutions), while multi-solitary-wave solutions of the equation were found numerically. In this talk, the multi-hump solutions of the equation will be formally derived. Twohump solutions are first derived based on the ideas from the rigorous existence proof of two-hump solutions. Then, three-hump solutions are formally obtained using a matching procedure. Finally, the ideas for formally deriving multi-hump solutions with arbitrary number of humps will be discussed. The ideas can be extended to derive multi-solitary waves on water of finite depth with small surface tension. (This is a joint work with Shengfu Deng)

Multi-scale dispersion from solutions to active particles

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Dispersion in waves and shear flows plays a crucial role in environmental mechanics and fluid physics, influencing both natural and engineered systems. Traditional Tay-lor-Aris dispersion theory, constrained by long-time asymptotic assumptions, fails to capture precisely the multi-scale dispersion dynamics during initial and transitional stages - limitations that become pronounced in complex flow environments. This study extends the Taylor-Aris theory by developing an analytical framework with unified spatio-temporal scales, generalizing dispersion from a description of unidirectional diffusion enhancement into a general methodology capable of characterizing the complete macroscopic transport of both passive solutes and active particles, thereby markedly broadening the applicability of the classical theory.

The equivalent pure-diffusion model formulated in the Lagrangian coordinate system demonstrates that the dispersion process is intrinsically a linear superposition of multi-scale diffusion, whose characteristics can be represented by an effective diffusivity tensor embodying spatial anisotropy and temporal nonlinear evolution. The asymptotic short-time particular solution derived herein for the first time reveals the key dynamics of the early-stage evolution of Taylor dispersion, establishing a new benchmark for the quantitative prediction of dispersion processes. By matching the short-time particular solution in moment space with the long-time Taylor dispersion solution, a multi-scale dispersion model bridging microscopic and macroscopic behaviour is constructed, effectively characterizing the evolution in the initial, transitional and asymptotic stages of dispersion.

With complete analytical solutions, we examine systematically dispersion in laminar and turbulent flows as well as active particle systems, and propose manipulation strategies for artificial particles. In classical Hagen-Poiseuille flows, we reconstruct a multi-scale dispersion model of a solute that overcomes traditional limitations during initial and transitional stages. The model's validity extends to log-law turbulent flows, where perturbation analysis identifies macroscopic information loss from cross-sectional averaging as the root cause of conventional model failures. For high-dimensional active particle systems, the developed multi-scale dispersion model reveals the physical origins of micro-algal hydrodynamic focusing: the transition from diffusive, ballistic to dispersive scaling results from the coupling between rotational dynamics and swim-induced dispersion. Notably, in the shear-rate transition region, we predict theoretically, for the first time, a transient effective trapping phenomenon. Furthermore, an externally controlled anisotropic dispersion model enables preferential accumulation and alignment of artificial particles, providing a theoretical basis for micro- and nano-robot design. Finally, we briefly demonstrate how free-surface wave and flow oscillation can be formulated in the continuum theory for both solutes and micro-swimmers, using Lagrangian Stokes drift and two-time-scale analysis. The results highlight the crucial roles of swimmers' taxes, morphology and wave-swimmer interactions in shaping vertical distributions and dispersion characteristics for motile phytoplankton.

This talk introduces three key theoretical contributions. First, the multi-scale dispersion framework in the Lagrangian coordinate system enables precise decoupling of dispersion. Second, the asymptotic short-time solution, combined with matching in the moment space, establishes a quantitative model for multi-scale dispersion. Third, the elucidation of wave motions and external field control strategies not only clarifies the evolution of hydrodynamic focusing and alignment but opens new avenues for intelligent transport manipulation.

These findings extend beyond high-dimensional dispersion-reaction systems to applications in colloidal rheology, pollutant migration and micro-fluidic device design, providing a theoretical foundation for quantitative analysis and intelligent regulation of environmental fluid mechanical systems.

Keywords: Dispersion, Multi-scale transport, Micro-organism dynamics, Coarse-grained dispersion model, Active particle transport

Dispersive effects on equatorial shallow-water shock waves

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The nonlinear dynamics of equatorial shallow-water waves are examined both theoretically and numerically. While unidirectional Kelvin waves are non-dispersive and can evolve into shocks, they also excite Poincaré waves which are dispersive. This dispersion has a significant impact on the dynamics of the shock. A reduced model has been developed to describe the behaviours of the Kelvin wave and the first Poincaré wave, incorporating an integral term that represents the effects of dispersion. The findings indicate that a sharp spike forms in the profile of the first Poincaré wave, resulting in an overshoot of the shock. Theoretical predictions are validated through numerical simulations of the primitive equations. This is joint work with Dong Shao and Zhan Wang (IMCAS).

Quasi-potential model for nonlinear wind-induced water waves and its application to air-sea interaction

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Wind-waves are one of the most common phenomena in the marine environment and have garnered significant attention since the last century. Numerous theoretical, experimental, and numerical studies have sought to clarify the complex dynamics. However, the interactions remain only partially understood due to the strong coupling, which complicates predictions and makes these interactions a challenging subject of study. This study presents an improved model for wind-driven water waves by incorporating both wind pressure and tangential viscous stress, which is referred to as quasi-potential theory model. Based on numerical simulations of turbulent flow over slowing moving wave boundaries, the wind pressure and tangential viscous stress are modelled as a phase-shifted function of wave elevation. A multi-scale asymptotic expansion gives the theoretical solution of the proposed system, where a crucial time-dependent parameter quantifying the wind effect on wave evolution appears, which can also be characterized as the growth rate. An unsteady numerical method based on a conformal mapping technique aligns the growth rate well with the theoretical prediction. Furthermore, the third-order statistical moments of wind-waves are established theoretically and show improved alignment with laboratory and marine observations in predicting wind-wave skewness and asymmetry.