Titles and Abstracts

of the conference

Geometric and analytic functional inequalities on discrete spaces

organized by Florian Fischer (U Bonn), Matthias Keller (U Potsdam), Bobo Hua (U Fudan) and Shubham Gupta (U Tsinghua) , Sep. 14 – 18, 2026

1. Davide Bianchi, School of Mathematics (Zhuhai), Sun Yat-sen University

Title: Existence, uniqueness and qualitative properties of solutions for evolution equations driven by accretive Laplacian-type operators on graphs

Abstract: We study evolution equations on infinite weighted graphs driven by accretive Laplacian-type operators, covering generalized porous medium equations and semilinear equations with monotone or Lipschitz nonlinearities. Accretivity (or ω-accretivity) on ℓ^p yields existence and uniqueness of mild solutions via Crandall–Liggett–Bénilan theory. The construction uses implicit Euler discretization and exhaustion by finite Dirichlet subgraphs, requiring no heat kernel or spectral information. We also establish a range of qualitative properties, including comparison principles, a priori estimates, positivity, and finite-time extinction.

2. Krzysztof Bogdan, Wrocław University of Science and Technology

Title: Discrete Hardy inequality for Sobolev-Bregman forms

Abstract: We establish ground-state representations and critical Hardy inequalities for Sobolev–Bregman forms in discrete settings. This is a joint project with Shubham Gupta, Kamil Kaleta, and Antoni Szczukiewicz.

3. Jonathan Breuer, The Hebrew University of Jerusalem.

Title: The Point Spectrum of Periodic Quantum Trees.

Abstract: Periodic Jacobi matrices on trees have been the subject of considerable interest in the past several years. Their quantum analogs, namely periodic quantum trees, have received much less attention and are less well understood. We consider the point spectrum of such operators, prove some analogous statements to those known in the discrete case and discuss some differences. This is joint work with Netanel Y. Levi.

4. Bartłomiej Dyda, Wrocław University of Science and Technology

Title: Fractional and non-fractional Hardy inequality on a lattice $\mathbb{Z}^d$

Abstract: I will show the following *pre*-Hardy inequality: if $sp < d$, then for some $K \in \mathbb{N}$ large enough,

$$\sum_{j \in \mathbb{Z}_+^d \setminus \{0\}} \frac{|u(j)|^p}{\|j\|_\infty^{sp}} \leq C(d,p,s,K) \sum_{n=1}^{\infty} \sum_{2^{n-1} \leq \|j\|_\infty < 2^n} \sum_{2^{n+K-1} \leq \|m\|_\infty < 2^{n+K}} \frac{|u(j)-u(m)|^p}{2^{(n+K)(d+sp)}}$$

for all functions u for which the left side is finite. Then I will show that it yields Hardy and fractional Hardy inequalities. The constants are explicit, but not optimal.

5. Huabin Ge, Renmin University of China

Title: Characterization of infinite ideal polyhedra in hyperbolic 3-space.

Abstract: In his seminal work, Rivin characterized finite ideal polyhedra in three-dimensional hyperbolic space. However, the characterization of infinite ideal polyhedra, as proposed by Rivin, has remained a long-standing open problem. In this paper, we introduce the combinatorial Ricci flow for infinite ideal circle patterns, a discrete analogue of Ricci flow on non-compact Riemannian manifolds, and prove a characterization of such circle patterns under certain combinatorial conditions. Our results provide affirmative solutions to Rivin's problem. This is joint work with Bobo Hua, Hao Yu and Puchun Zhou.

6. Michael Hinz, Bielefeld University

Title: On first order operators on (metric) graphs and fractals

Abstract: While "second order differential operators" on (certain) fractals have now been understood for quite some time, an understanding of "first order differential operators" and first order equations on fractals is still very recent. We will give a gentle introduction from a (metric) graph perspective and then present recent results, joint with Waldemar Schefer (Bielefeld). They generalize earlier results of Hino, Strichartz and others and led to novel integration by parts formulas and to first well-posedness results for equations of continuity type on fractals.

7. Paul Horn, University of Denver

Title: Bakry-Emery Curvature, local pseudo-randomness, and applications

Abstract: In recent years, various notions of discrete curvature have been developed in order to translate powerful ideas from Riemannian geometry to the study of graphs. For many of these, their power arises because of deep connections between discrete Laplace operators and the Laplace-Beltrami operator on Riemannian manifolds. In a sense, many consequences of curvature lower bounds are 'local certificates' of properties known to be certified more globally by graph spectra. One important use of Laplacian spectra within combinatorics where an analogue is more subtle deals with random-like properties of a graph: It is well known that the spectra of the Laplacian certifies pseudo-random properties of the edge set. Here were discuss the connection of Bakry-Emery curvature to a 'local pseudo-randomness', and give some applications – both exploiting the pseudo-randomness to understand combinatorial

properties of the underlying graph and to using pseudo-randomness to give bounds on combinatorial curvature.

8. Qi Hou, Beijing Institute of Mathematical Sciences and Applications

Title: Heat kernels of left-invariant diffusions on infinite dimensional compact groups

Abstract: Infinite dimensional compact (connected metrizable) groups are projective limits of compact Lie groups. Many of them arise as infinite products, such as products of circles. On such product groups, the heat kernel measures of many bi-invariant Laplacians are the laws of independent Brownian motions on the factors. The Kakutani dichotomy theorem for product type measures thus applies, and satisfactory characterizations of such heat kernels have been obtained, pointing to a variety of properties ranging from genuine infinite dimensional phenomena to finite dimensional ones. Contrary to this diagonal case, the heat kernels of left-invariant Laplacians or sub-Laplacians are not well-understood, because most standard tools in finite dimensions do not carry over. I plan to present some attempts to analyze the heat kernel behaviors for left-invariant elliptic or subelliptic diffusions, via comparisons of intrinsic distances or Dirichlet forms at various levels. I will try to discuss the relations with graphs. This talk is based on joint works with Laurent Saloff-Coste.

9. Xia Huang, East China Normal University

Title: Hardy-Rellich inequalities on $\mathbb{Z}^d$

Abstract: In this talk, we will show the sharp asymptotic behaviour of the corresponding optimal constants in discrete Hardy-Rellich inequality on the lattice $\mathbb{Z}^d$ as $d \to \infty$ with $\Delta^{\ell/2}$, for any $\ell \geq 1$. Our approach is based on general weighted identities, which yield anisotropic weighted inequalities on the d-dimensional torus, then settle the higher order cases by iteration.

10. Xueping Huang, School of Mathematics and Statistics, Nanjing University of Information Science and Technology

Title: Sharp criteria for the existence of positive solutions to Lane–Emden-type inequalities

Abstract: We study positive solutions of the superlinear Lane-Emden inequality $-\Delta u \geq \sigma u^q$, $q > 1$, on infinite locally finite weighted graphs and connected domains. We explore both the potential theoretic approach and a new method via flow decomposition.

We first prove that solvability is equivalent to a nonlinear Green potential inequality. Then under conditions ensuring regular Green function estimates, we prove sharp characterizations of existence of positive solutions.

Our main result gives a sharp criterion of the form

$$\sum_{n\geq 1} \frac{n^{2q-1}}{\mu(B(o,n))^{q-1}} = \infty,$$

for the nonexistence of positive solution in full generality. Intrinsic metrics can be employed for general positive potential function σ. The proof combines flow decomposition and Hardy estimates along paths. Applications to concrete examples are also discussed. This talk is based on joint work with Qingsong Gu, Lu Hao, and Yuhua Sun.

11. Kamil Kaleta, Wrocław University of Science and Technology

Title: Discrete evolution confined by a potential

Abstract: We study a discrete-time Markov evolution on a countably infinite state space describing the motion of a particle confined by an unbounded potential. Probabilistically, the model is given by a Markov chain whose trajectories are killed at a rate determined by the potential. The resulting non-conservative transition semigroup may be viewed as a discrete-time analogue of the classical Feynman–Kac semigroup. Our main interest lies in long-range Markov chains generated by nonlocal discrete operators.

In this talk, I will give an introduction to the subject and discuss several results from our recent work. These include sharp estimates of ground states and discrete-time heat kernels, characterizations of intrinsic ultra- and hypercontractivity properties, and their applications. In contrast to the continuous-time setting, where hypercontractivity is characterized by logarithmic Sobolev inequalities, we identify a different family of functional inequalities that completely characterizes hypercontractivity in discrete time. I will also compare our findings with the corresponding theory for nearest-neighbour random walks on locally finite graphs.

A central role is played by the *direct step property* (DSP), which provides a simple and robust framework for the analysis. It yields short proofs of many results and applies to a remarkably broad class of processes and operators. I will discuss several constructions leading to DSP and illustrate them with concrete examples. Connections with non-local discrete Schrödinger operators, including fractional powers of graph Laplacians and related pseudo-relativistic operators, will also be presented.

This is joint ongoing work with Wojciech Cygan (Wrocław), René Schilling (Dresden), and Mateusz Śliwiński (Lublin).

12. Supanat Kamtue, Chulalongkorn University, Thailand

Title: Entropic Ricci Curvature on Graphs: An Introduction and Survey

Abstract: The notion of entropic Ricci curvature introduced by Erbar and Maas provides a synthetic framework for lower Ricci curvature bounds on finite graphs and Markov chains based on optimal transport and entropy. In this talk, I will present the basic construction of this curvature notion and discuss some of its analytic consequences, particularly its connections to functional inequalities. I will conclude with a brief survey of recent developments, including explicit curvature estimates for several families of graphs.

13. David Krejčiřík, Czech Technical University in Prague

Title: Hardy inequalities for magnetic p-Laplacians

Abstract: We establish improved Hardy inequalities for the magnetic p-Laplacian due to adding nontrivial magnetic fields. In particular, for Aharonov-Bohm potentials, we show that the sharp constant in the Hardy inequality becomes strictly larger than in the case of a magnetic-free p-Laplacian. We also post some remarks with open problems.

The talk is based on joint work with Cristian Cazacu, Nguyen Lam and Ari Laptev published in Nonlinearity.

14. Shiping Liu, University of Science and Technology of China

Title: Spectral rigidity of the hypercube via Bakry–Émery curvature

Abstract: Hypercube graphs are fundamental model spaces of positive curvature in discrete geometry. In this talk, we will discuss spectral rigidity results for hypercube graphs in the category of unweighted graphs, from the viewpoint of Bakry–Émery curvature. In particular, we show that the hypercube is forced by a weaker spectral multiplicity assumption than previously known, and that this improvement is optimal. The proof combines a global spectral embedding associated with the first eigenspace with a local analysis of curvature matrices. This is based on joint work with Yanlong Ding and Chiyu Zhou.

15. Jun Masamune, Kyoto University

Title: Emergence of an unsaturated Darcy's law and a Richards-type equation via homogenization of the Stokes-Cahn-Hilliard system

Abstract: We study the homogenization of a microscopic Stokes–Cahn–Hilliard system in a periodically perforated porous medium with logarithmic free energy. The model describes an incompressible two-phase flow with diffuse interfaces inside pore-scale geometries and incorporates capillary effects through the chemical potential. We prove that the microscopic system converges to a Darcy–Korteweg type law coupled with a generalized Richards-type equation. A central consequence of the analysis is that the resulting macroscopic law generally cannot be reduced to the classical unsaturated Darcy law. In particular, the effective driving mechanism is not, in general, governed by a scalar gradient structure, reflecting the influence of interfacial energy and microscopic phase configurations. This is a collaborative effort with Lei RONG (Tohoku University).

16. Florentin Münch

Title: TBA

Abstract: TBA

17. Noema Nicolussi, University of Innsbruck

Title: Optimal eigenvalues on a metric graph with densities

Abstract: Motivated by the notion of conformal eigenvalues for surfaces, we investigate Laplacians on a metric graph with varying mass density. This setting provides a common framework for several well-studied classes of operators: discrete Laplacians,

Dirichlet-to-Neumann operators on graphs, and Kirchhoff Laplacians (a.k.a. quantum graph operators). Our main interest is the spectral optimization problem for eigenvalues with respect to the underlying mass density, which turns out to behave rather differently than in the manifold setting. In particular, we discuss the relation of optimal eigenvalues with geometrical properties, including a complete geometric description of the first optimal eigenvalue and a Weyl law.

Based on joint work with Kiyan Naderi (University of Innsbruck).

18. Norbert Peyerimhoff, Durham University, UK

Title: A survey about some curvature rigidity results

Abstract: Two widely used Ricci curvature notions for general graphs are Ollivier and Bakry-Émery curvature. They are defined on the edges and vertices of a graph and they can be computed with very reasonable effort. A lower positive curvature bound for these local curvature values imply bounds for global invariants like the diameter and the first non-zero Laplace eigenvalue. These relations are motivated by similar results in the setting of Riemannian manifolds (Bonnet-Myers and Lichnerowicz), and it is natural to ask whether all spaces, for which these curvature/diameter and curvature/eigenvalue inequalities hold with equality, can be classified. Since these weaker strictness inequality assumptions lead to strong restrictions of the spaces, such results are called rigidity results. The aim of this talk is to survey about the current state those rigidity results.

19. Felix Pogorzelski, Leipzig University

Title: News on Hardy inequalities for discrete Laplacians

Abstract: Hardy inequalities play a fundamental role in analysis and its applications. Although the earliest forms of these inequalities, established by G. H. Hardy and others around 1920, were discrete in nature, subsequent research focused predominantly on the continuum setting for many decades. More recently, the discrete perspective has attracted renewed attention, in particular through the study of Hardy weights and their optimality properties for Laplace- and Schrödinger operators on graphs. In this talk, we focus on some new developments and emerging ideas.

20. Marcel Schmidt, Friedrich-Schiller-Universität Jena

Title: Criticality theory for nonlinear Dirichlet forms

Abstract: Nonlinear Dirichlet forms are lower semicontinuous convex functionals on L^2-spaces whose subdifferentials generate order-preserving and L^∞-contractive nonlinear semigroups. Important examples include Cheeger energies on metric measure spaces, the p-Dirichlet energy, and energies arising from nonlinear resistance networks.

In this talk, we present a criticality theory for such functionals. We study the dichotomy between subcriticality and criticality, characterize subcriticality in terms of weighted Hardy inequalities and the finiteness of the associated Green operator, and characterize criticality through the existence of a ground state in the sense of Agmon. Our main aim is to identify the additional assumptions under which such a theory can be developed and to highlight both the similarities and the differences

between the linear and nonlinear settings. Motivated by the theme of the conference, particular emphasis will be placed on applications to discrete spaces.

This is joint work with Ian Zimmermann (Jena).

21. Alberto G. Setti, Università dell'Insubria

Title: On the accretivity and m-accretivity of Laplacians and porous medium-type operators on graphs

Abstract: We study the accretivity and m-accretivity of Laplacian and porous medium-type operators on weighted graphs, and give several conditions that imply these properties for maximal operators and investigate when these operators agree with various restrictions. For porous medium-type operators on ℓ^1 and for Laplacians on ℓ^p for $p \in [1, \infty)$, we show that there always exists a dense subset of the domain on which the maximal operator is m-accretive. Under additional conditions on the graph, we then prove that the maximal operators are m-accretive on the entire domain, and not just a dense subset. For the ℓ^2 case, we give connections to Markov uniqueness and essential self-adjointness. For the ℓ^∞ case, we establish the equivalence of stochastic completeness at infinity, m-accretivity for the maximal Laplacian on ℓ^∞ and m-accretivity of the minimal Laplacian on ℓ^1.

This is joint work with Davide Bianchi (Sun Yat-sen University, Zhuhai, China), Matthias Keller (Universität Potsdam, Germany) and Radosław K. Wojciechowski (York College and Graduate Center, CUNY, New York, USA).

22. František Štampach, Czech Technical University in Prague
and Jakub Waclawek, Czech Technical University in Prague (Tandem Talk)

Title: Optimal Hardy inequalities for powers of the discrete Laplacian on the half-line

Abstract: The talk consists of two parts. In the first part (presented by F. Š.), we discuss optimal discrete p-Hardy–Rellich–Birman inequalities for general $p > 1$, i.e., optimal Hardy-type inequalities for an arbitrary *integer power* of the discrete p-Laplacian on the half-line. In the second part (presented by J. W.), we discuss similar developments, partly still in progress, for $p = 2$ and an arbitrary *positive power* of the discrete Laplacian on the half-line.

23. Melchior Wirth, Leipzig University

Title: Wasserstein-like metrics and gradient flow of the entropy

Abstract: It was first realized by Jordan, Kinderlehrer and Otto that the classical heat flow can be viewed as gradient flow of the differential entropy in Wasserstein space. This result was subsequently extended to cover a number of heat-like flows, including the heat flow on Riemannian manifolds with lower Ricci curvature bound (Erbar) and the flow generated by the graph Laplacian on finite graphs (Maas, Mielke), where an adapted transport metric instead of the Wasserstein metric has to be used. It was shown by the speaker that this construction can be extended to all Markov semigroups under suitable technical assumptions. In this talk I will focus on the case of infinite graphs. I will discuss different metric versions of gradient flows and

I will give sufficient conditions for weighted graph Laplacians to generate the gradient flow of the relative entropy with respect to an adapted Wasserstein-like metric.

24. Radosław K. Wojciechowski, York College and the Graduate Center of the City University of New York

Title: Essential self-adjointness, form uniqueness and stochastic completeness for symmetric graphs

Abstract: We present a series of results, many achieved with undergraduate students, on the questions of essential self-adjointness, form uniqueness and stochastic completeness on graphs. In particular, we give characterizations in terms of geometric quantities for these properties in the case of birth–death chains and, if possible, weakly spherically symmetric graphs. The main technical tool is a study of α-harmonic functions which can be reduced to working with recurrence relations for these graphs. This includes joint work with Luis Hernandez, Atsushi Inoue, Sean Ku, Jun Masamune, Genevieve Romanelli and Emily Shteynberg.

25. Yunyan Yang, School of Mathematics, Renmin University of China

Title: Evolution of weights on graphs and its application on network analysis

Abstract: In this talk, we concern the evolution of weights on weighted graph, including various Ricci curvature flows, entropy flow and density flow. Also, we demonstrate the performance of these flows in community detection and classification tasks for graph neural networks.

26. Dong Zhang, Peking University

Title: The eigenvalue problem for infinity-Laplacian on finite graphs

Abstract: In this talk, I will present some results on the graph infinity-Laplacian eigenproblem, including nodal domain theorems, as well as eigenvalue estimates involving the packing radii, the independence number, and the matching number.

27. Liang Zhao, Beijing Normal University

Title: Curvature of hypergraphs with applications in hypergraph community detection

Abstract: We first introduce definitions of Ollivier and Lin-Lu-Yau curvatures of hypergraphs. Next, we define a hypergraph Ricci flow and prove the long-time existence of the flow. As applications, we focus on community detection in hypergraphs, which is both instrumental for functional module identification and intricate due to higher-order interactions among nodes of hypergraphs. We develop a Ricci-flow-based community detection approach (HyperRCD) that deforms hyperedge weights through curvature-driven evolution. Extensive experiments on both synthetic and real-world hypergraphs demonstrate that HyperRCD exhibits competitive performance across diverse datasets.

The report is based on collaborative works with Tian Yulu, Ma Jicheng and Yang Yunyan.