April 6, 2026 - Monday

Localisation of symmetric PDE eigenvalues

Carsten Carstensen
Humboldt-Universität zu Berlin

Recent advances in the nonconforming FEM approximation of symmetric PDE eigenvalue problems include guaranteed lower eigenvalue bounds (GLB) and their adaptive finite element computation. While the first generation of post-processed GLB from nonconforming FEM [CG14a, CG14b] is unconditional and hence allows a verified eigenvalue localisation even on coarse meshes, it is incompatible with adaptive mesh refinement if the eigenfunction localises.

The second generation of GLB [CEP21, CP23, CP24] circumvent this difficulty with new extrastabilized schemes that directly compute approximations as GLB. An adaptive mesh-refining algorithm for the effective eigenvalue computation for the Laplace and bi-Laplace operator with optimal convergence rates in terms of the number of degrees of freedom is highlighted.

The talk concludes with an introduction to skeletal schemes [CGP25, CGZ26] for higher-order GLB, in particular for fourth-order problems. This enable a high-precision localisation of eigenvalues with rates comparable to adaptive Argyris FEM approximations [CG24].

The presentation concerns model examples for the Laplace, bi-Laplace, and Schrödinger eigenvalue problem [CS26] and is based on recent joint work with Tim Stiebert and former students.

References

[1] C. Carstensen, A. Ern, and S. Puttkammer. Guaranteed lower bounds on eigenvalues of elliptic operators with a hybrid high-order method. Numerische Mathematik, 149(2):273-304, 2021.

[2] C. Carstensen and D. Gallistl. Guaranteed lower eigenvalue bounds for the biharmonic equation. Numerische Mathematik, 126(1):33-51, 2014.

[3] C. Carstensen and J. Gedicke. Guaranteed lower bounds for eigenvalues. Mathematics of Computation, 83(290):2605-2629, 2014.

[4] C. Carstensen and B. Gräßle. Rate-optimal higher-order adaptive conforming FEM for biharmonic eigenvalue problems on polygonal domains. Computer Methods in Applied Mechanics and Engineering, 425:116931, 2024.

[5] C. Carstensen, B. Gräßle, and E. Pirch. Comparison of guaranteed lower eigenvalue bounds with three skeletal schemes. Computer Methods in Applied Mechanics and Engineering, 433:117477, 2025.

[6] C. Carstensen, B. Gräßle, and Q. Zhai. Higher-order guaranteed lower biharmonic eigenvalue bounds. Computer Methods in Applied Mechanics and Engineering, 451:118617, 2026.

[7] C. Carstensen and S. Puttkammer. Direct guaranteed lower eigenvalue bounds with optimal a priori convergence rates for the bi-Laplacian. SIAM Journal on Numerical Analysis, 61(2):812-836, 2023.

[8] C. Carstensen and S. Puttkammer. Adaptive guaranteed lower eigenvalue bounds with optimal convergence rates. Numerische Mathematik, 156(1):1-38, 2024.

[9] C. Carstensen and T. Stiebert. Old and new Schrödinger eigenvalue localisation. Submitted to SISC, 2026.

[10] C. Carstensen, Q. Zhai, and R. Zhang. A skeletal finite element method can compute lower

eigenvalue bounds. SIAM Journal on Numerical Analysis, 58(1):109-124, 2020.

Optimal and robust finite element methods for strain gradient elasticity

Xuehai Huang
Shanghai University of Finance and Economics

Two optimal and robust finite element methods for strain gradient elasticity are shown in this talk. The first one is an interior penalty nonconforming finite element method using a newly designed quadratic H^1 nonconforming element for displacement discretization. The second one is a low-order mixed finite element method with a third-order tensor, which discretizes the third-order stress gradient tensor by a low-order H (div) tensor element and the displacement by the linear Crouzeix-Raviart element. Both methods achieve optimal convergence rates, and the error estimates are robust with respect to the Lamé coefficient λ and the size parameter ι. Numerical results are presented to validate the theoretical findings.

Highly efficient and energy stable Multi-step SAV approaches for phase field models

Yanping Chen
Nanjing University of Posts and Telecommunications

Recently, the scalar auxiliary variable (SAV) approach and its extended SAV-based approaches have been widely used to simulate a series of phase field models. However, many SAV-based schemes are known for the stability of a 'modified' energy. In this paper, we construct a series of modified SAV approaches with unconditional energy dissipation law based on several improvements to the traditional SAV approach. Firstly, by introducing the three-step technique, we can reduce the number of constant coefficient linear equations that need to be solved at each time step, while retaining all of its other advantages. Secondly, the addition of energy-optimized technique and SAV/Lagrange multiplier technique can make the numerical schemes have the advantage of preserving the original energy dissipation. Thirdly, we use the first-order approximation of the energy balance equation in the GSAV approach, instead of discretizing the dynamic equation of the auxiliary variable, so that we can construct the high-order unconditional original energy stable numerical schemes. Finally, representative numerical examples show that the efficiency and accuracy of the proposed schemes are improved.

From HDG to NURBS-CutHDG: Exact Geometry Treatment in High-Fidelity Hybridisable Discontinuous Galerkin

Matteo Giacomini
Universitat Politècnica de Catalunya

Hybrid discretisation methods have emerged as a versatile framework for the numerical simulation

of flow and coupled multi-physics problems (e.g., fluid-structure interaction). The hybridisable discontinuous Galerkin (HDG) paradigm naturally allows to devise both low-order methods such as the face-centred finite volume (FCFV) [1] robust to mesh distortion and stretching, and high-order discretisations with reduced dissipation and dispersion errors [2]. The resulting approximations are stable across a wide range of regimes, from compressible, to weakly compressible and perfectly incompressible flows.

In the first part of this talk, we will present some recent developments in HDG formulations for flow problems, with special focus on the accurate treatment of convection phenomena via Riemann solver-inspired stabilisations [3] and the coupling of discontinuous and continuous Galerkin for multi-physics problems [4]. In particular, we will discuss the interplay of high-fidelity functional approximation and accurate geometry representation [5]. Achieving high-order accuracy in domains with complex boundaries typically requires body-fitted curved meshes, whose generation remains a major bottleneck in computer-aided engineering, especially in three dimensions and in problems with moving/evolving interfaces.

To overcome this limitation, we then introduce a paradigm for unfitted high-order HDG, embedding NURBS boundaries into simple, Cartesian background meshes [6]. This leads to exact boundary representations cutting through the computational mesh. In this context, accurate numerical integration in cut elements is achieved through the NURBS-enhanced finite element method (NEFEM) paradigm [7], and boundary conditions are imposed in a weak sense, consistently with standard HDG [8]. The resulting NURBS-CutHDG formulation retains the favorable properties of classical HDG methods, eliminating the need for body-fitted, curved, high-order meshes, and without introducing additional globally-coupled unknowns.

Numerical experiments will be presented to showcase the accuracy, ro-bustness, and stability of the discussed HDG methods, with single- and multi-physics systems involving fluids in different flow regimes, highlighting their potential for high-fidelity simulations on complex geometries without the burden of curved, body-fitted meshes.

References

[1] R. Sevilla, M. Giacomini, and A. Huerta, "A face-centred finite volume method for second-order elliptic problems", International Journal for Numerical Methods in Engineering, 115(8):986-1014, 2018.

[2] M. Giacomini, R. Sevilla, and A. Huerta, "HDGlab: An open-source implementation of the hybridisable discontinuous Galerkin method in MATLAB", Archives of Computational Methods in Engineering, 28(3):1941-1986, 2021.

[3] J. Vila-Pérez, M. Giacomini, R. Sevilla, and A. Huerta, "Hybridisable discontinuous Galerkin formulation of compressible flows", Archives of Computational Methods in Engineering, 28(2):753-784, 2021.

[4] A. La Spina, M. Kronbichler, M. Giacomini, W. A. Wall, and A. Huerta, "A weakly compressible hybridizable discontinuous Galerkin formulation for fluid-structure interaction problems", Computer Methods in Applied Mechanics and Engineering, 372:113392, 2020.

[5] M. Giacomini and R. Sevilla, "Discontinuous Galerkin approximations in computational mechanics: hybridization, exact geometry and degree adaptivity", *SN* Applied Sciences, 1:1047, 2019.

[6] S. Piccardo, M. Giacomini, and A. Huerta, "An unfitted high-order HDG method for two-fluid Stokes flow with exact NURBS geometries", Journal of Computational Physics, 512:113143, 2024.

[7] R. Sevilla, S. Fernández-Méndez, and A. Huerta, "NURBS-enhanced finite element method (NEFEM)", International Journal for Numerical Methods in Engineering, 76(1):56-83, 2008.

[8] B. Cockburn, J. Gopalakrishnan, and R. Lazarov, "Unified Hybridization of Discontinuous

Galerkin, Mixed, and Continuous Galerkin Methods for Second Order Elliptic Problems", SIAM Journal on Numerical Analysis, 47(2):1319-1365, 2009.

An ALE sliding interface method for fluid-structure interaction with a rotating rigid structure

Jiashun Hu
The Hong Kong Polytechnic University

Sliding interfaces offer an effective approach for incompressible fluid-structure interaction with a rigid body undergoing large rotation, but they pose challenges because the fluid meshes on the two sides of the sliding interface are nonmatching (and may even overlap in an isoparametric setting). In this talk, we present an ALE sliding-interface formulation based on a rotational ALE description and a skew-symmetric Nitsche stabilization on an artificial sliding interface, yielding an energy-dissipating method. We outline a first-order fully discrete scheme that preserves this dissipation mechanism, and briefly discuss an inf-sup stability result for the interface coupling and its implication for unique solvability. Numerical examples in 2D and 3D-including cylindrical and spherical sliding inter-faces-demonstrate robustness under rotation, convergence, and energy dissipation.

Maxwell compactness and polytopal methods

Simon Lemaire
INRIA

Real-world multiphysics problems are often nonlinear. As such, their analysis (existence of solutions to the PDE model, convergence of numerical approximations) often relies on compactness arguments. At the continuous level, for PDE models based on the de Rham complex, these compactness results are well established, and due to Rellich (for H^1) and to Weck/Weber (for $\boldsymbol{H}(\mathbf{curl})$ and $\boldsymbol{H}(\mathrm{div})$). The last two results are often referred to, in the literature, as Maxwell compactness, as they naturally appear in the study of models in electromagnetism. At the discrete level, even for conforming finite element approximations on standard meshes, Maxwell compactness results do not directly follow from their infinite-dimensional counterpart. This is even more true for polytopal methods, which hinge on fully algebraic spaces and discrete operator and potential reconstructions. In this talk, we will discuss Maxwell compactness results in the case of two different polytopal methods, namely the HHO (Hybrid High-Order) and the DDR (Discrete De Rham) approaches. The former is non-conforming and non-compatible, whereas the latter is conforming and compatible. The material of this talk is based on joint works (i) with S. Pitassi (Univ. Montpellier) on the one hand, and (ii) with T. Chaumont-Frelet (INRIA Lille) and J. Droniou (CNRS, Univ. Montpellier) on the other hand.

Preasymptotic error estimates of higher-order EEM for the time-harmonic Maxwell equations with large wave number

Haijun Wu
Nanjing University

The time-harmonic Maxwell equations with impedance boundary condition and large wave number are discretized using the second-type Nédélec's edge element method (EEM). Preasymptotic error bounds are derived, showing that, under the mesh condition $\kappa^{2p+1}h^{2p}$being sufficiently small, the error of the EEM of order p in the energy norm is bounded by $\mathcal{O}\left(\kappa^{p}h^{p}+\kappa^{2p+1}h^{2p}\right)$, while the error in the κ-scaled $\boldsymbol{L}^2$ norm is bounded by $\mathcal{O}\left((\kappa h)^{p+1}+\kappa^{2p+1}h^{2p}\right)$. Here, κ is the wave number and $h$is the mesh size. Numerical tests are provided to illustrate our theoretical results.

Stochastic Interacting Particle Field Methods in the Computation of Chemotaxis and Haptotaxis PDEs

Zhiwen Zhang
The University of Hong Kong

In this talk, I will present the latest developments in stochastic interacting particle field (SIPF) methods. The methodology originates from the Lagrangian framework and is used to compute the asymptotic behaviors of PDEs, such as effective diffusivities and KPP front speeds. We then turn to interacting particle methods for elliptic-type Keller–Segel equations. Very recently, we have further extended our research on SIPF methods, enabling the computation of parabolic-type Keller–Segel and haptotaxis equations. If time permits, I will also discuss how these results connect with the field of generative modeling.

April 7, 2026 - Tuesday

High-order Structure-Preserving Schemes for Special Relativistic Hydrodynamics

Huazhong Tang
Peking University

This talk introduces two high-order accurate structure-preserving finite difference schemes for the special relativistic hydrodynamics (RHD). The first is the physical-constraints-preserving (PCP) scheme, which preserves the positivity of the rest-mass density and the pressure and the bounds of the fluid velocity and is built on the local Lax-Friedrichs (LxF) splitting, the WENO reconstruction, the PCP flux limiter, and the high-order strong stability preserving time discretization. The key to developing such scheme is to prove the convexity and other properties of the admissible state set and to discover a concave function with respect to the conservative vector. The second is the entropy stable (ES) scheme, whose semi-discrete version satisfies the entropy inequality. The key is to technically construct the affordable entropy conservative (EC) flux of the semi-discrete secondorder accurate EC schemes satisfying the semi-discrete entropy equality for the found convex entropy pair. As soon as the EC flux is derived, the dissipation term can be added to give the semi-discrete ES schemes satisfying the semi-discrete entropy inequality. The WENO reconstruction for the scaled entropy variables and the previous time discretization are implemented to obtain the fully-discrete high-order "ES" schemes. The performance of the proposed schemes has been demonstrated by numerical experiments. By the way, we also briefly review other relative works on the structure-preserving schemes for the special RHDs. Those works have been further to the general equation of state and the special relativistic magnetohydrodynamics etc., see our papers listed below for details.

Continuous and discontinuous Galerkin methods for elliptic problems in nondivergence form

Omar Lakkis
University of Sussex

I will outline established and more recent techniques for the discretization of equations of the form $A(x) : D^2u(x) + b(x) \cdot Du(x) - c(x)u(x) = f(x)$ with Dirichlet/Neumann/oblique boundary conditions. Our approaches involve a discrete recovery of the discrete solution's Hessian, which is not a function. I will emphasise recent work using a least-squares approach to do so. I will present applications to fully nonlinear elliptic problems and optimal transport.

A posteriori error estimates for layer solutions

Natalia Kopteva
University of Limerick

Solutions of singularly perturbed partial differential equations typically exhibit sharp boundary and interior layers, as well as corner singularities. To obtain reliable numerical approximations of such solutions in an efficient way, one may want to use meshes that are adapted to solution singularities using a posteriori error estimates. In this talk, we shall discuss residual-type a posteriori error estimates singularly perturbed reaction-diffusion equations and singularly perturbed convection-diffusion equations, The error constants in the considered estimates are independent of the diameters of mesh elements and of the small perturbation parameter. Note also that some of the presented results are new and interesting when applied to classical elliptic equations.

I will start by revisiting some earlier work, including a posteriori error estimates on anisotropic meshes. In the final part of the talk, more recent, current, and future work will be discussed

References

[1] A. Demlow, S. Franz, N. Kopteva (2023). Maximum norm a posteriori error estimates for convection-diffusion problems. IMA J. Numer. Anal., 43, 2562-2584.

[2] A. Demlow, N. Kopteva (2016). Maximum-norm a posteriori error estimates for singularly perturbed elliptic reaction-diffusion problems. Numer. Math., 133, 707-742.

[3] Z. Dong, E. H. Georgoulis, N. Kopteva (2025). Adaptive robust interior penalty discontinuous Galerkin methods. submitted.

[4] N. Kopteva (2015). Maximum-norm a posteriori error estimates for singularly perturbed reaction-diffusion problems on anisotropic meshes. SIAM J. Numer. Anal., 53, 2519-2544.

Parallel orbital-updating method for electronic structure calculations and the related numerical analysis

Xiaoying Dai
Chinese Academy of Sciences

The Kohn–Sham density functional theory method is a powerful, widely used method for computation of ground state electronic energies and densities in chemistry, materials science, biology, and nanoscience. Its key mathematical model is the Kohn-Sham equation, which is a nonlinear differential operator eigenvalue problem. In this talk, I will introduce our parallel orbital-updating approach for the Kohn-Sham equation and the related numerical analysis.

A posteriori error estimation and adaptivity for the finite element discretization of second-order PDE problems set in unbounded domains

Theophile Chaumont-Frelet
INRIA

This talk is dedicated to the discretization of second-order PDE problems set in unbounded domains. The approximation procedure for such problems is typically divided into two steps. First (i), a modeling error is introduced by truncating the domain at a finite distance L from the origin. Then (ii), the remaining bounded domain is meshed and the corresponding finite element space is used in the discretization. Standard a posteriori error estimators for this problem only take into account the error incurred in step (ii), but disregard the modeling error introduced in step (i).

I will discuss an alternative viewpoint where the whole domain is discretized by an infinite mesh, on which a finite-dimensional finite element space of degree is constructed. The construction of the finite element space implicitly involves a truncation procedure, so that the situation is formally identical to the standard setting. However, the key insight of this interpretation is that one can construct an error estimator that accounts for all the sources of error induced by the discretization. This estimator can in particular steer adaptive mesh refinement algorithms that automatically adjust the truncation of the domain.

I will rigorously show that the proposed estimator is reliable and efficient, and that the corresponding adaptive algorithm converges at optimal rates. I will also present numerical examples illustrating these theoretical findings.

Part of these results are based on a joint work with Gregor Gantner.

A hierarchical approach for multicontinuum homogenization in high contrast media

Yin Yang
Xiangtan University

A recently developed upscaling technique, the multicontinuum homogenization method, has gained significant attention for its effectiveness in modeling complex multiscale systems. This method defines multiple continua based on distinct physical properties and solves a series of constrained cell problems to capture localized information for each continuum. However, solving all these cell problems on very fine grids at every macroscopic point is computationally expensive, which is a common limitation of most homogenization approaches for non-periodic problems. To address this challenge, we propose a hierarchical multicontinuum homogenization framework. The core idea is to define hierarchical macroscopic points and solve the constrained problems on grids of varying resolutions. The local solutions are decomposed into the linear interpolation of contributions inherited from preceding levels and an additional correction term. This combination is substituted into the original constrained problems, and the correction term is resolved using finite element (FE) grids of varying sizes, depending on the level of the macropoint. By normalizing the computational cost of fully resolving the local problem to $\mathcal{O}(1)$, we establish that our approach incurs a cost of $\mathcal{O}\left(L\eta^{(1-L)d}\right)$, highlighting substantial computational savings across hierarchical layers L, coarsening factor η, and spatial dimension d. Numerical experiments validate the effectiveness of

the proposed method in media with slowly varying properties, underscoring its potential for efficient multiscale modeling.

Adaptive FEM with explicit time integration for the wave equation

Marcus Grote
University of Basel

Starting from a recent a posteriori error estimator for the finite element solution of the wave equation with explicit time-stepping [Grote, Lakkis, Santos, 2024], we devise a space-time adaptive strategy which includes both time evolving meshes and local time-stepping [Diaz, Grote, 2009] to overcome any overly stringent CFL stability restriction on the time-step due to local mesh refinement. Moreover, at each time-step the adaptive algorithm monitors the accuracy thanks to the error indicators and recomputes the current step on a refined mesh until the desired tolerance is met; meanwhile, the mesh is coarsened in regions of smaller errors. Leapfrog based local time-stepping is applied in all regions of local mesh refinement to incorporate adaptivity into fully explicit time integration with mesh change while retaining efficiency. Numerical results illustrate the optimal rate of convergence of the a posteriori error estimators on time evolving meshes. This is joint work mit Omar Lakkis (University of Sussex) and Carina Santos (University of Basel).

Learning-based Linear Solvers for Multiphysics Problems

Chensong Zhang
Chinese Academy of Sciences

Solving large sparse linear systems is the main computational bottleneck in multiphysics simulation. Traditional iterative solvers face a fundamental trilemma, struggling to combine efficiency, robustness, and usability. This talk presents a novel framework that breaks this deadlock by merging multilevel solvers with data-driven learning. We will outline our preliminary steps to break the trilemma, including the optimization of multilevel components and new preconditioners for coupled PDE systems. By automating solver parameter tuning within a modular software architecture, our work tries to pave the way for intelligent, adaptive, and scalable solver technology for next-generation scientific simulations.

Chen-Song Zhang, PhD. Graduated from the Applied Mathematics \& Scientific Computing program at the University of Maryland, College Park, US; Worked as a postdoctoral fellow at the Penn State University, University Park, US; Currently working at the Academy of Mathematics and Systems Science, CAS, China. Main research interests include numerical analysis, adaptive methods, petroleum reservoir simulation, and complex fluid/flow simulation.

A generalized Hessian-based error estimator for an IPDG formulation of the biharmonic problem in two dimensions

Lorenzo Mascotto
University of Milano Bicocca

We consider a two dimensional biharmonic problem and its discretization by means of a symmetric interior penalty discontinuous Galerkin method. A novel split of an error measure based on a generalized Hessian into two terms measuring the conformity and nonconformity of the scheme is proven. This splitting is the departing point for the design of a new error estimator, which is provably reliable and efficient for polynomial degree larger than 3, and does not involve any DG stabilization. Such an error estimator can be bounded from above by the standard DG residual error estimator. Numerical results assess the theoretical predictions, including the efficiency of the proposed estimator, for all polynomial degrees larger than or equal to 2 . This is joint work with Théophile Chaumont-Frelet (Inria-Lille) and Joscha Gedicke (University of Bonn).

April 8, 2026 -Wednesday

Physics-Preserving Numerical Modeling for Fluid Flow in Porous Media: From Pore-Scale Dynamics to Field-Scale Multiphysics Applications

Shuyu Sun
Tongji University

Modeling fluid flow in porous media, spanning pore-scale dynamics to field-scale applications, is fundamental to geophysical and engineering endeavors such as carbon sequestration, enhanced oil recovery, underground gas storage, and subsurface contaminant transport. This talk integrates two interconnected research thrusts: thermodynamically consistent numerical frameworks for field-scale multiphase/multicomponent flow in deformable media, and robust pore-scale algorithms in digital rock physics (DRP) that enhance reservoir characterization and enable accurate upscaling. At the field scale, we present two tailored examples addressing key challenges in coupled fluid-solid systems. For multicomponent gas flow in poroelastic media, the framework integrates the Maxwell-Stefan-Darcy model for precise diffusion description, Biot's poroelasticity theory for solid deformation, and the Peng-Robinson equation of state for thermodynamic rigor. For immiscible two-phase flow in poro-viscoelastic media, we leverage fluid and solid free energies to characterize capillarity and skeleton viscoelasticity, adhering rigorously to the second law of thermodynamics via an energy dissipation law. A stabilized discretization strategy based on adaptive time-stepping, mixed finite elements, and discontinuous Galerkin methods eliminates non-physical solutions and ensures energy stability, local mass conservation, and strict boundedness of molar densities or phase saturations. Complementing these field-scale models, our pore-scale Digital Rock Physics (DRP) research provides critical insights for reservoir characterization, especially in tight formations, fractured media, and vuggy rocks. Key pore-scale contributions include: 1) a novel pore network model (PNM) derived from model reduction, controlling errors relative to more computationally expensive direct numerical simulation (DNS); 2) a pixel-free flashlight search medial axis algorithm for efficient PNM parameter extraction; 3) an SPH-based particle method for pore-scale flow that preserves mass, momentum, and energy dissipation; and 4) a two-scale molecular dynamics-Monte Carlo (MD-MC) framework for simulating tight pore phenomena (e.g., shale gas/oil transport and hydrogen storage leakage). These pore-scale tools address limitations of conventional DNS and enable accurate prediction of core properties such as absolute/relative permeability, capillarity, and effective elastic parameters. Numerical experiments across scales validate the robustness, efficiency, and physical fidelity of the proposed frameworks, demonstrating their capability to handle heterogeneous media, complex phase interactions, dynamic fluid-solid coupling, and microscale effects. Together, these methods form a comprehensive toolkit for simulating real-world porous media processes where thermodynamic consistency, computational stability, and cross-scale integration are paramount.

Inf-sup theory for quasi-static poroelastic models

Pietro Zanotti
University of Milan

Quasi-static poroelastic models describe the flow of a fluid inside an elastic porous medium. Such

models have attracted growing attention in recent years because of their wide range of applications. The most well-established techniques for both the analysis and the discretization of these problems build either on the so-called Faedo-Galerkin method or on the theory of implicit evolution equations. The talk introduces and motivates an alternative technique, based on the inf-sup theory. The proposed approach, in contrast to previous ones, aims at establishing an isomorphism between the solution and the data spaces, which is a desirable property for the design, the stability and the error analysis of discretization schemes. Advantages and current limitations of this approach are illustrated by means of new results and open problems. This is a joint work with A. Khan (IIT Roorkee) and C. Kreuzer (TU Dortmund).

Staggered Raviart-Thomas DG methods on polygonal meshes

Eun-Jae Park
Yonsei University

The Staggered Discontinuous Galerkin (SDG) method is a class of finite element methods designed to solve partial differential equations while preserving local conservation properties and handling complex geometries. It employs a staggered mesh structure in which scalar and vector variables are discretized on distinct, yet overlapping, primal and dual meshes. This arrangement facilitates a natural enforcement of conservation laws and enables element-wise postprocessing for superconvergent approximations. The SDG framework supports high-order accuracy and geometric flexibility, making it well-suited for problems involving unstructured or polytopal meshes. Moreover, by decoupling the discretization of variables, the method enhances stability and allows for efficient hybridization, yielding compact global systems and connections to other modern finite element approaches such as HHO , WG, and virtual element methods.

In this talk, we present a new family of polygonal SDG methods utilizing Raviart-Thomas mixed finite element spaces. Formulated in a mixed setting, the method approximates the primal and dual variables using a locally H^1-conforming finite element space and a locally $H(div)$-conforming Raviart-Thomas finite element space, respectively. Unlike in classical mixed FEM, the standard $RT_k \times P_k$ pair is not inf-sup stable in the SDG framework due to the staggered primal-dual mesh structure. To restore stability, the primal space is enriched with bubble type functions on dual elements. The inf-sup stability and optimal convergence are proved. Next, with a simple modification of the loading term we are able to obtain globally $H(div)$-conforming velocity fields. The theoretical results are verified by numerical experiments. Some recent work on the eigenvalue problem will be discussed.

Numerical analysis of PDE-driven surface evolution: Geometric flow, free boundary, fluid-structure interaction

Buyang Li
The Hong Kong Polytechnic University

In this talk, we present recent advances in the numerical analysis of solution-driven surface evolution, spanning geometric flows, two-phase flows, and fluid-structure interactions. We report a matrix-vector framework for deriving rigorous error estimates of finite element approximations

to an evolving surface driven by the solutions of PDEs on the surface itself, as well as interfaces driven by fluids on both sides or coupled with elastic structures. Our analysis addresses the interplay between errors in computing interface location and PDE solutions, providing error bounds for the problem with unknown surface/interface evolution. The rigorous error analysis allows us to understand the stability of numerical algorithms and to design more stable algorithms to preserve high-quality meshes during computations, including artificial tangential motions on the evolving surfaces and interfaces, and artificial mesh velocity which improves the bulk meshing for fluid free-boundary problems. These methods enhance the accuracy and efficiency in simulating complex interfacial dynamics.

Metric-Driven Numerical Methods for Variational Eigenvalue Problems

Daniel Peterseim
University of Augsburg

We introduce the concept of metric-driven numerical methods for the numerical solution of constrained minimization problems and the associated Euler-Lagrange equations, including nonlinear PDE eigenvalue problems. The central idea is to formulate iterative schemes by defining gradients with respect to problem-adapted metrics, so that the choice of metric directly determines the numerical method and its convergence behavior. Interestingly, the choice of metric also induces approximation spaces with enhanced properties, in particular for problems with limited regularity or heterogeneous coefficients. From this perspective, we recover the Localized Orthogonal Decomposition (LOD) as a well-known class of approximation spaces derived from a new viewpoint. We apply the approach to the computation of ground states of spin-orbit coupled Bose-Einstein condensates.

This is joint work with P. Henning and L. Huynh (Bochum).

Stability and error analysis of fully discrete original energy-dissipative and length-preserving scheme for the Landau-Lifshitz-Gilbert equation

Xiaoli Li
Shandong University

The Landau-Lifshitz-Gilbert (LLG) equation, regarded as a gradient flow with manifold constraint, is the fundamental model describing magnetization dynamics in ferromagnetic materials. In this talk, we first construct a linear and fully discrete finite difference numerical scheme, based on the projection method for the LLG equation, which is capable of simultaneously preserving the non-convex manifold constraint and an unconditional original energy dissipation. In the error analysis, the classical theoretical technique becomes ineffective, due to the presence of the nonlinear Laplacian term, which in turn poses a significant challenge. We rewrite the numerical method in an equivalent weak form by using a point-wise length preserving feature and derive an optimal error estimate. Some numerical experiments are presented to verify the theoretical findings and illustrate the robustness and effectiveness of the proposed method.

TBA

Andreas Veeser
University of Milan

TBA

Analysis of high-contrast Stokes interface problems with multiple subdomains

Chupeng Ma
Great Bay University

We consider a stationary Stokes problem with a piecewise constant viscosity coefficient. We identify a sufficient and necessary condition on the coefficient under which the continuous and discrete inf-sup conditions (with standard finite element pairs) are robust with respect to the jump of the coefficient. We also discuss robust FE error estimates and preconditioning techniques for the coefficients that do not satisfy this condition.

Accelerating Crystalline Defect Simulations via Systematic Higher-Order Far-Field Boundary Conditions

Yangshuai Wang
National University of Singapore

fundamental challenge in the atomistic simulation of crystalline defects, such as point defects and dislocations, is the slow convergence of standard supercell methods due to long-range elastic fields. This presentation introduces a rigorous mathematical and numerical framework for constructing systematically improvable, higher-order boundary conditions (BCs) that effectively minimize domain-size effects. We first characterize the discrete elastic far-field using a low-rank decomposition consisting of discrete multipole terms and continuum predictors. For point defects, we present an iterative "moment iteration" scheme that approximates multipole tensors within finite domains, achieving accelerated convergence in both geometry and energy norms without significant computational overhead. Extending this to dislocations, we address the added complexity of core singularities by employing a spectral Galerkin method combined with rescaled variables. Numerical experiments on Tungsten (BCC) for various defect types, including vacancies, interstitials, and screw/edge dislocations, validate our theoretical error estimates. The results demonstrate that first-order and second-order BCs can achieve high accuracy on relatively small computational domains, offering a robust pathway for efficient high-accuracy multiscale material modeling.

April 9, 2026 -Thursday

A Minimal-Deformation-Rate Framework for Surface Evolution: From Curvature Flows to Shape Optimization

Rong Tang
The Hong Kong Polytechnic University

Numerical simulations of evolving surfaces, such as mean curvature flow and shape optimization, often suffer from mesh deterioration under large deformations. In this talk, I will present a numerical framework based on the Minimal Deformation Rate (MDR) strategy, whose main purpose is to preserve mesh quality throughout the evolution without remeshing.

First, for curvature-driven flows, including mean curvature flow and surface diffusion on both closed and open surfaces with moving contact lines, we design a new BGN-MDR scheme by exploiting the key observation that the Barrett-Garcke-Nürnberg (BGN) and MDR formulations differ only in one degree of freedom, thereby combining the discrete energy stability of the classical BGN approach with the mesh-quality preservation of MDR.

Second, for PDE-constrained shape optimization and surface hole filling, we couple a second-order inertial flow with MDR, which both accelerates convergence in flat energy landscapes and avoids frequent remeshing during large geometric deformations through a suitable MDR-based mesh-motion strategy.

A weak SPH method for parabolic partial differential equations

Xiaoping Xie
Sichuan University

We develop a new Smoothed Particle Hydrodynamics (SPH) method, termed weak SPH (wSPH), for solving d-dimensional $2m$th order parabolic partial differential equations. Unlike traditional SPH methods that directly discretise PDEs in strong form with forward Euler time stepping, the WSPH method employs the backward Euler scheme for temporal discretisation and formulates the problem in a weak (variational) form. We establish an error estimation framework for wSPH by decomposing the total numerical error into approximation error, numerical quadrature error, and optimisation error, each of which admits explicit bounds in terms of the time step τ, the number of particles N and the number of quadrature points NS. Our analysis shows that with B-spline smoothing kernels, wSPH achieves a spatial convergence rate of order $N^{-\left(\frac{1}{2}+\frac{1}{d+1}\right)}$ and a temporal convergence rate of order τ, where d denotes the spatial dimension. We further derive an optimal scaling relation between N and NS that balances the spatial discretisation error and the quadrature error. Numerical experiments are provided to verify the theoretical results.

Robust Finite Element Methods for Homogeneous and Heterogeneous Strain Gradient Elasticity

Pingbing Ming
Chinese Academy of Sciences

We shall discuss the robust finite element methods for the strain gradient elasticity. Both the homogeneous and heterogeneous media will be covered. The convergence of the proposed elements is independent of the material parameters. This is a joint work of Yulei Liao (National University of Singapore).

Arbitrary order approximations at constant cost for Timoshenko beam network models

Andreas Rupp
Saarland University

This presentation considers the numerical solution of Timoshenko beam network models, comprised of Timoshenko beam equations on each edge of the network, which are coupled at the nodes of the network using rigid joint conditions. Through hybridization, we can equivalently reformulate the problem as a symmetric positive definite system of linear equations posed on the network nodes. To discretize the beam network model, we propose a hybridizable discontinuous Galerkin method that can achieve arbitrary orders of convergence under mesh refinement without increasing the size of the global system matrix. As a preconditioner for the typically very poorly conditioned global system matrix, we employ a two-level overlapping additive Schwarz method. We also investigate convergence of the corresponding preconditioned conjugate gradient method under appropriate connectivity assumptions on the network.

Efficient and physics-preserving algorithms for flows in porous media

Huangxin Chen
Xiamen University

Modeling and simulation of flows in porous media are of great interest in the fields of hydrology and petroleum reservoir engineering. In this talk we will introduce an efficient enriched Petrov-Galerkin (EPG) method for flows in porous media, including the applications of EPG method for flows in fractured porous media. We will also introduce thermodynamically consistent mathematical models for non-isothermal flows in porous media, and the physics-preserving algorithms will be discussed.

Analysis of a finite element method for second order uniformly elliptic PDEs in non-divergence form

Weifeng Qiu
City University of Hong Kong

We propose one finite element method for both second order linear uniformly elliptic PDE in non-divergence form and the uniformly elliptic Hamilton-Jacobi-Bellman (HJB) equation. For both linear elliptic PDE in non-divergence form and the HJB equation, we prove the well-posedness of strong solutions in $W^{2,p}(\Omega)$ and optimal convergence in discrete $W^{2,p}$-norm of the finite element approximation to the strong solution for $1 < p \leq 2$ on convex polyhedra in $\mathbb{R}^d (d = 2, 3)$. If the domain is a two-dimensional non-convex polygon, p is valid in a more restricted region. Furthermore, we relax the assumptions on the continuity of coefficients of the HJB equation, which have been widely used in literature.

Upwind Finite Element Methods for General Convection-Diffusion Equations

Shuonan Wu
Peking University

This talk presents upwind-type stabilized finite element methods for convection-diffusion equations, focusing on recent extensions to vector-valued problems in H(curl)spaces. While exponential fitting provides an alternative stabilization strategy by incorporating boundary-layer characteristics, the discussion emphasizes upwind methods due to their adaptability in convection-dominated regimes. Two discontinuous Galerkin (DG) approaches—a primal DG and a hybridizable DG (HDG)—leverage weighted residual formulations and parameterized numerical traces to ensure stability and accuracy in magnetic advection-diffusion problems. Furthermore, other classical scalar techniques are generalized to H(curl)settings: a streamline upwind/Petrov-Galerkin (SUPG) method incorporates residual-based stabilization and a discrete advection operator, and a local projection stabilization (LPS) scheme enriches approximation spaces with H(curl)-conforming bubbles to satisfy local inf-sup conditions. These advances illustrate concrete approaches for designing and analyzing upwind-type discretizations for H(curl)convection-diffusion problems, demonstrating how classical scalar methodologies are adapted to the distinctive mathematical structure of vector field spaces.

Nonconforming approximation of Ginzburg-Landau-type semilinearities

Lara Théallier
Humboldt-Universität zu Berlin

The Landau-de Gennes model for nematic liquid crystals provides computational challenges within a non-convex minimization problem. The associated Euler-Lagrange equations form a semilinear second-order elliptic boundary value problem with reduced regularity in non-convex domains as well as additional topological singularities called vortices The energy landscape in this non-convex

minimisation problem is unexpectedly rich with many stationary points of the energy functional and severe difficulties for the local solve. Nonconforming Crouzeix-Raviart finite elements allow for lower energy bounds in the asymptotic range of sufficiently fine meshes.

The presentation departs with 2D computational benchmark examples and explains the origin of vortex singularities for Dirichlet data of non-zero winding number for larger Ginzburg parameters ℓ for this is a novel aspect in the mandatory adaptive mesh-refining. A priori existence of discrete solutions and their weak or strong (global) convergence towards stationary points along subsequences is illuminated. An asymptotic a priori and a posteriori local error analysis with optimal rates for appropriate adaptive algorithms concludes the presentation.

References

[1] C. Carstensen, A. K. Dond, R. R. Maity, N. Nataraj, L. Théallier: Adaptive Crouzeix-Raviart FEM for a non-convex Ginzburg-Landau model for nematic liquid crystals, CMAME (2026) 118146.

April 10, 2026 -Friday

Discrete duality relations in the error analysis of a hybridizable method for convex minimization problems

Ngoc Tien Tran
University of Augsburg

This talk establishes duality relations on the discrete level for a class of hybridizable methods for convex minimization problems. The application to the error analysis leads to error estimates for the energy error with convergence rates under sufficient smoothness assumptions. Furthermore, the explicitly known maximizer on the dual level directly provides degrees of freedom for a construction of a conforming stress approximation, allowing guaranteed error control for the energy error via duality gap.

An efficient mass-conservative and bound-preserving limiting technique and its application on the neutron transport equation

Zuodong Wang
Eastern Institute of Advanced Study

A limiting technique for scalar transport equations is presented. The originality of the method is that it does not require solving nonlinear optimization problems nor does it rely on the construction of a low-order approximation. The method has minimal complexity and is numerically demonstrated to maintain high-order accuracy. The performance of the method is illustrated on the neutron transport equation.