Representation type of Hopf algebras with dual Chevalley property

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3. Results and some new Hopf algebras

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  From representation theoretic point of view, he proved the following result:

Theorem

(1) A finite algebraic group $\mathcal{G}$ is of finite representation type iff $H(\mathcal{G}) := \mathcal{O}(\mathcal{G})^*$ is a Nakayama algebra;
(2) If $\mathcal{G}$ is tame, then $H(\mathcal{G})$ is special biserial.

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- We test them through using Hopf algebras with (dual) Chevalley property.

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- H has Chevalley property if and only if H^* has dual Chevalley property.

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$H_{2,K}$ is generated by c, b, x, y with relations:

- $c^2 = 1, \;\; b^2 = 1, \;\; x^2 = \frac{1}{2}(1 + c + b - cb), \;\; cb = bc, \;\; xc = bx, \;\; xb = cx,$
- $y^2 = 0, \;\; yc = -cy, \;\; yb = -by, \;\; yx = \sqrt{-1}cxy.$

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The coalgebra and antipode are given by:

- $\Delta(c) = c \otimes c, \ \ \varepsilon(c) = 1, \ \ S(c) = c,$
- $\Delta(b) = b \otimes b, \ \ \varepsilon(b) = 1, \ \ S(b) = b,$
- $\Delta(x) = \frac{1}{2}(x \otimes x + bx \otimes x + x \otimes cx - bx \otimes cx), \ \ \varepsilon(x) = 1, \ \ S(x) = x,$
- $\Delta(y) = c \otimes y + y \otimes 1, \ \ \varepsilon(y) = 0, \ \ S(y) = -c^{-1}y.$

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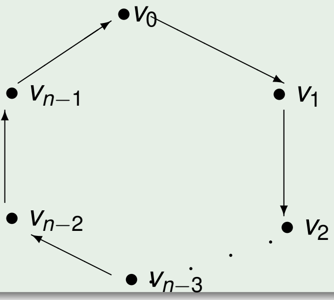

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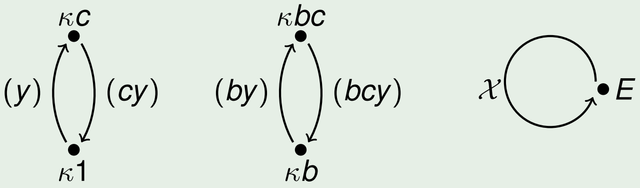
The dual Gabriel quiver of above $H_{2,k}$ is shown below:



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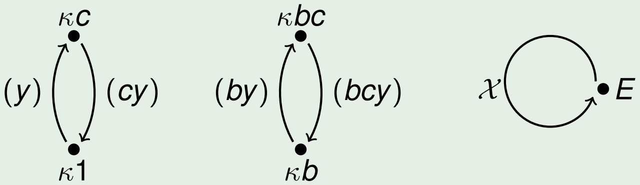

$H_{2,k}$ is neither elementary nor pointed.

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Theorem

Let H be an elementary Hopf algebra, then
(i) H is of finite representation type if and only if $n_H = 0$ or $n_H = 1$;
(ii) If H is tame, then $n_H = 2$;
(iii) If $n_H \geq 3$, then H is of wild type.

Outline

Multiplicative and primitive matrices

Definition

Let (H, Δ, ε) be a coalgebra over κ.

(1) A square matrix $\mathcal{G} = (g_{ij})_{r\times r}$ over H is said to be multiplicative, if for any $1 \leq i, j \leq r$, we have $\Delta(g_{ij}) = \sum_{t=1}^{r} g_{it} \otimes g_{tj}$ and $\varepsilon(g_{ij}) = \delta_{i,j}$, where $\delta_{i,j}$ denotes the Kronecker notation;

(2) A multiplicative matrix $\mathcal{C}$ is said to be basic, if its entries are linearly independent.

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- The set of all the simple subcoalgebras of H is denoted by $\mathcal{S}$, which corresponds a complete set of basic multiplicative matrices.

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Let (H, Δ, ε) be a coalgebra over κ. Suppose $\mathcal{C} = (c_{ij})_{r\times r}$ and $\mathcal{D} = (d_{ij})_{s\times s}$ are basic multiplicative matrices over H.

(1) A matrix $\mathcal{X} = (x_{ij})_{r\times s}$ over H is said to be $(\mathcal{C}, \mathcal{D})$-primitive, if

$$\Delta(x_{ij}) = \sum_{k=1}^{r} c_{ik} \otimes x_{kj} + \sum_{t=1}^{s} x_{it} \otimes d_{tj}$$

holds for any $1 \leq i, j \leq r$;

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Multiplicative and primitive matrices

- Denote ${}^{1}\mathcal{S} = \{C \in \mathcal{S} \mid \kappa 1 + C \neq \kappa 1 \wedge C\}$. For any $C \in {}^{1}\mathcal{S}$, we can fix a complete family $\{\mathcal{X}_C^{(\gamma_C)}\}_{\gamma_C \in \Gamma_C}$ of non-trivial $(1, \mathcal{C})$-primitive matrices.

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- Denote

$$^1\mathcal{P} := \bigcup_{C \in {}^1\mathcal{S}} \{\mathcal{X}_C^{(\gamma_C)} \mid \gamma_C \in \Gamma_C\}.$$

Finite type and tame type

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(1) *H is of finite corepresentation type if and only if* $| {}^1\mathcal{P} | = 1$ *and* $\dim_\kappa(C) = 1$, *where* $C \in {}^1\mathcal{S}$.

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   (i) $|\,{}^1\mathcal{P}\,| = 2$ *and for any* $C \in {}^1\mathcal{S}$, $\dim_\kappa(C) = 1$;

   (ii) $|\,{}^1\mathcal{P}\,| = 1$ *and* $\dim_\kappa(C) = 4$, *where* $C \in {}^1\mathcal{S}$.

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(3) *If one of the following holds, H is of wild corepresentation type.*

   (i) $|{}^1\mathcal{P}| \geq 3$;
   (ii) $|{}^1\mathcal{P}| = 2$ *and there exists some* $C \in {}^1\mathcal{S}$ *such that* $\dim_\kappa(C) \geq 4$;
   (iii) $|{}^1\mathcal{P}| = 1$ *and* $\dim_\kappa(C) \geq 9$, *where* $C \in {}^1\mathcal{S}$.

Finite representation type

Corollary

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Finite representation type

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Let H be a finite dimensional Hopf algebra with Chevalley property. Then the following are equivalent:

- *H is of finite representation type;*
- *H is a Nakayama algebra;*
- *$\Gamma(H)$ is a disjoint union of basic cycles.*

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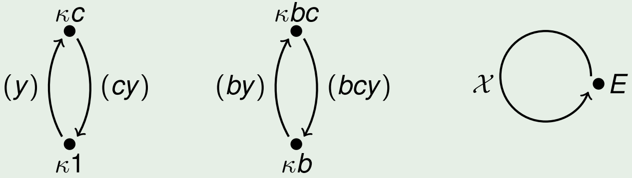
- This suggests the following conjecture:
  Conjecture. Let H be a finite dimensional Hopf algebra. Then H is of finite representation type if and only if H is a Nakayama algebra.

Example

- $H_{2,k}$ is of finite corepresentation type since its dual Gabriel quiver is



Tame type-Example

Example

- Let H_{32} be the Hopf algebra of dimensional 32 which is generated by z, y, t, p_1, p_2 satisfying the following relations:
  - $z^2 = 1, \ \ y^2 = 1, \ \ t^2 = 1, \ \ zy = yz, \ \ tz = zt, \ \ ty = yt,$
  - $zp_1 = p_1 z, \ \ yp_1 = p_1 y, \ \ tp_1 = -p_1 t, \ \ zp_2 = p_2 z, \ \ yp_2 = p_2 y, \ \ tp_2 = -p_2 t,$
  - $p_1^2 = \lambda(1 - z), \ \ p_2^2 = -\lambda(1 - z), \ \ p_1 p_2 + p_2 p_1 = 0.$

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    The coalgebra structure and antipode are given by:
  - $\Delta(z) = z \otimes z, \ \ \Delta(y) = y \otimes y, \ \ \varepsilon(z) = \varepsilon(y) = 1,$
  - $\Delta(t) = \frac{1}{2}\left[(1+y)t \otimes t + (1-y)t \otimes zt\right], \ \ \varepsilon(t) = 1,$
  - $S(z) = z, \ \ S(y) = y, \ \ S(t) = \frac{1}{2}\left[(1+y)t + (1-y)zt\right],$
  - $\Delta(p_1) = p_1 \otimes 1 + \frac{1}{2}(1+z)\, t \otimes p_1 + \frac{1}{2}(1-z)\, yt \otimes p_2,$
  - $\Delta(p_2) = p_2 \otimes 1 + \frac{1}{2}(1+z)\, yt \otimes p_2 + \frac{1}{2}(1-z)\, t \otimes p_1.$

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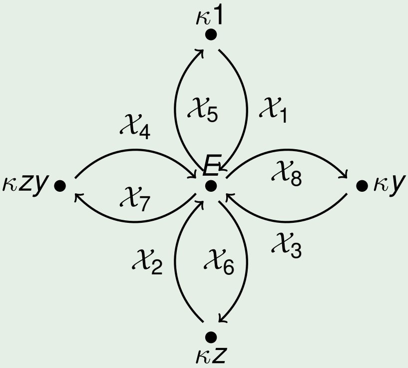

It is apparent that H is of infinite corepresentation type and in fact tame.

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Let κ be an algebraically closed field of characteristic 0 and H a finite-dimensional Hopf algebra over κ with Chevalley property. Then $\mathrm{gr}(H)$ *is of tame type if and only if*

$$\mathrm{gr}(H) \cong (\kappa\langle x, y\rangle / I) \times H/J_H$$

for ideal I which is one of the following forms:

(1) $I = (x^2 - y^2, yx - ax^2, xy)$ for $0 \neq a \in \kappa$;

(2) $I = (x^2, y^2, (xy)^m - a(yx)^m)$ for $0 \neq a \in \kappa$ and $m \geq 1$;

(3) $I = (x^n - y^n, xy, yx)$ for $n \geq 2$;

(4) $I = (x^2, y^2, (xy)^m x - (yx)^m y)$ for $m \geq 1$.

Remark and conjecture

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- $o(a) = i(a)$ for $a \in Q(H)_0$;
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Discrete corepresentation type

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- So if H is of finite dimensional, then H is of discrete corepresentation type if and only if it is of finite corepresentation type by Brauer-Thrall Theorem.

Discrete corepresentation type

Theorem

Let H be a non-cosemisimple Hopf algebra over κ with the dual Chevalley property and $H_{(1)}$ be its link-indecomposable component containing $\kappa 1$. If the coradical of $H_{(1)}$ is finite dimensional,

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(1) *H is of discrete corepresentation type;*

(2) *Every vertex in $\mathrm{Q}(H)$ is both the start vertex of only one arrow and the end vertex of only one arrow, that is, $\mathrm{Q}(H)$ is a disjoint union of basic cycles;*

(3) *There is only one arrow $C \to \kappa 1$ in $\mathrm{Q}(H)$ whose end vertex is $\kappa 1$ and* $\dim_\kappa(C) = 1$*;*

(4) *There is only one arrow $\kappa 1 \to D$ in $\mathrm{Q}(H)$ whose start vertex is $\kappa 1$ and* $\dim_\kappa(D) = 1$*.*

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Theorem

Let H be a non-cosemisimple Hopf algebra over κ with the dual Chevalley property of discrete corepresentation type and $H_{(1)}$ be its link-indecomposable component containing $\kappa 1$. Denote ${}^1\mathcal{S} = \{C \in \mathcal{S} \mid \kappa 1 + C \neq \kappa 1 \wedge C\}$. If the coradical of $H_{(1)}$ is infinite-dimensional,

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(1) $|{}^1\mathcal{P}| = 1$ *and* ${}^1\mathcal{S} = \{\kappa g\}$ *for some* $g \in G(H)$*;*

(2) $|{}^1\mathcal{P}| = 2$ *and* ${}^1\mathcal{S} = \{\kappa g, \kappa h\}$ *for some different group-like elements* g, h*;*

(3) $|{}^1\mathcal{P}| = 1$ *and* ${}^1\mathcal{S} = \{C_k\}$ *for some* $C_k \in \mathcal{S}$ *with* $\dim_\kappa(C_k) = 4$.

Discrete corepresentation type

Remark

Note that in the above theorem, cases (1) and (2) imply that $H_{(1)}$ is pointed which already was considered before. One of our contributions is to show that the case (3) indeed occurs.

Discrete corepresentation type

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Example

As an algebra, $H(e_{\pm 1}, f_{\pm 1}, u, v)$ is generated by u, v, e_i, f_i for $i \in \mathbb{Z}$, subject to the following relations

$$1 = e_0 + f_0, \quad e_i e_j = e_{i+j}, \quad f_i f_j = f_{i+j}, \quad e_i f_j = f_j e_i = 0,$$
$$e_i u = (-1)^i u e_i, \quad f_i u = (-1)^i u f_i, \quad e_i v = (-1)^i v e_i, \quad f_i v = (-1)^i v f_i,$$
$$u^2 = v^2 = 0, \quad uv = -vu,$$

for any $i, j \in \mathbb{Z}$.

Discrete corepresentation type

Example

The comultiplication, counit and the antipode are given by

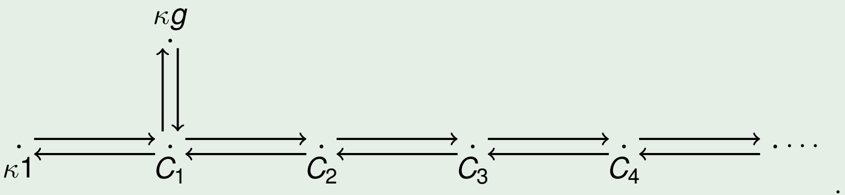
$$\Delta(e_i) = e_i \otimes e_i + f_i \otimes f_{-i}, \quad \varepsilon(e_i) = 1, \quad S(e_i) = e_{-i},$$
$$\Delta(f_i) = e_i \otimes f_i + f_i \otimes e_{-i}, \quad \varepsilon(f_i) = 0, \quad S(f_i) = f_i,$$
$$\Delta(u) = 1 \otimes u + u \otimes e_1 + v \otimes f_{-1}, \quad \varepsilon(u) = 0, \quad S(u) = -vf_{-1} - ue_{-1},$$
$$\Delta(v) = 1 \otimes v + u \otimes f_1 + v \otimes e_{-1}, \quad \varepsilon(v) = 0, \quad S(v) = -uf_1 - ve_1,$$

for any $i \in \mathbb{Z}$.

Discrete corepresentation type

Example

The link quiver of $H(e_{\pm 1}, f_{\pm 1}, u, v)$ is of the following form:



Further new Hopf algebras

- In above example, the coradical H_0 is indeed a kind of abelian extension.

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A Hopf extension

$$K \xrightarrow{\iota} H \xrightarrow{\pi} A$$

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Definition

A Hopf extension

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is called abelian if A is cocommutative and K is commutative.

- Let G, F be finite groups and κ^G denote the dual Hopf algebra of κG. Abelian extensions

$$\kappa^G \xrightarrow{\iota} H \xrightarrow{\pi} \kappa F$$

of κF by κ^G were classified by Masuoka(2002), and the above H can be expressed as $\kappa^G \#_{\sigma,\tau} \kappa F$.

Further new Hopf algebras

- We can consider abelian extension with F maybe infinite.

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Example

Let $\mathbb{Z}_2 = \{g \mid g^2 = 1\}$. Define group actions $\mathbb{Z}_2 \overset{\triangleleft}{\leftarrow} \mathbb{Z}_2 \times \mathbb{Z} \overset{\triangleright}{\rightarrow} \mathbb{Z}$ on the sets by

$$1 \triangleleft i = 1, \quad g \triangleleft i = g, \quad 1 \triangleright i = i, \quad g \triangleright i = -i,$$

for any $i \in \mathbb{Z}$. Consider the case when σ and τ are trivial, that is,

$$\sigma(i, j) = 1$$

and

$$\tau(x) = 1 \otimes 1$$

for any $i, j \in \mathbb{Z}$ and $x \in \mathbb{Z}_2$. In such a case, let $H(\mathbb{Z}, \mathbb{Z}_2)$ be the abelian extension. Then $H(\mathbb{Z}, \mathbb{Z}_2)$ is indeed the coradical of the Hopf algebra in the previous Example.

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Example

Let $G = \mathbb{Z}_{2n} = \{g \mid g^{2n} = 1\}$ for some $n \geq 1$. Define group actions $\mathbb{Z}_{2n} \overset{\triangleleft}{\leftarrow} \mathbb{Z}_{2n} \times \mathbb{Z} \overset{\triangleright}{\rightarrow} \mathbb{Z}$ on the sets by

$$g^i \triangleright j = (-1)^i j, \quad g^i \triangleleft j = g^i,$$

for any $1 \leq i \leq 2n$, $j \in \mathbb{Z}$. Consider the case when σ and τ are trivial, that is, $\sigma(i,j) = 1$ and $\tau(x) = 1 \otimes 1$ for any $i, j \in \mathbb{Z}$ and $x \in \mathbb{Z}_{2n}$. In such a case, let $H(\mathbb{Z}, \mathbb{Z}_{2n})$ be the abelian extension. Then

$$H(\mathbb{Z}, \mathbb{Z}_{2n}) \cong \kappa\mathbb{Z}_{2n} \oplus \Big(\bigoplus_{j \in \mathbb{Z}_+} \bigoplus_{k=0}^{n-1} E_j^{(k)}\Big).$$

Further new Hopf algebras

- We get the following general observation.

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Proposition

All abelian extensions

$$\kappa^G \xrightarrow{\iota} H \xrightarrow{\pi} \kappa F$$

with G being finite are cosemisimple.

Thank you for your attention!